Mating Markets

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Introduction

The economic analysis of the "market for marriage" has a long tradition, marked by the seminal contributions of Becker (1973, 1974). Two more recent developments have made it the focus of renewed interest: new models of household behavior, and a class of tractable specifications for econometric work. These two threads have converged to generate richer predictions and empirical applications. The collective approach to household behavior (Chiappori, 1988, 1992) has emphasized the importance of decision power within the household. Equilibrium in the marriage market clearly plays an important role in determining how it is allocated; in turn, matching models provide a natural and powerful tool to analyze marriage markets.

Indeed, the various models we shall survey share a common, basic structure. First, marriage (or cohabitation) generates economic and non economic gains, relative to singlehood. Second, these gains are couple-specific; they depend on the characteristics of both partners. Third, and as a consequence, individuals de facto compete on the marriage market. The intrahousehold allocation of resources, decision power, and ultimately well-being within couples thus formed is endogenously codetermined by the nature of this competition. Broadly speaking, intrahousehold allocation can be seen as a price that clears the corresponding market. Finally, this allocation is in turn reflected in the decision-making process within the household: it influences both pre- and post-marital decisions. In particular, human capital investments change both the amount of surplus and its allocation; this plays a role in individual choices of an education. Conversely, the resulting balance of power within the couple may influence decisions regarding labor supply, consumption and savings, or investment into children.

These general ideas can obviously be implemented in many different ways. A large fraction of this survey will be devoted to a particular class of models, based on frictionless matching. To a large extent, this feature simply reflects the current state of the literature; most existing models of the marriage market use a frictionless framework. One reason for this is parsimony. Just as standard price theory starts from simple supply/demand models, frictionless models are a natural point of departure. As with price theory, many central insights can be analyzed within a parsimonious framework. Empirical considerations are another reason to focus on frictionless models. Often only matching patterns are observable. As we shall see, then even the simplest empirical matching models require strong functional form assumptions on the stochastic distributions of the shocks. More complex models, involving for instance search frictions, have additional parameters and accordingly require additional assumptions that are hard to validate: the nature of the search technology, or the distributions of the various stochastic processes governing arrivals and exits. A natural conjecture is that given only data about matching patterns in a cross-section, it is in fact impossible to distinguish between models involving frictions and frictionless models with unobserved heterogeneity[1]. While more work is needed on that topic, we believe that Occam's razor should favor the simpler settings.

Within the frictionless family, models with perfectly Transferable Utility play a central role. In these models, utility can be transferred within couples at a constant "exchange rate"[2]. This makes them relatively simple and tractable. They imply that equilibria always maximize total surplus, a property that considerably simplifies both theoretical analysis and empirical implementation. The theory of matching markets with Transferable Utility was initiated by Koopmans and Beckmann (1957) and its central results were obtained by Shapley and Shubik (1972). Empirical analysis took a long time to catch up, however. As we will explain in Section 2, the presence of unobserved heterogeneity on the two sides of the market complicates the estimation of matching models. The contribution of Choo and Siow (2006) opened the door to the specification of a class of tractable and flexible models. They have been used to analyze matching patterns, as well as their impact on pre-marital investments and post-marital decisions.

Transferable Utility models, however, have a core weakness: they imply that each household behaves as a single decision maker, whose preferences do not depend on the environment[3]. The price to pay is that the household's aggregate behavior—consumptions and labor supply—is not affected by changes in relative earnings of men and women, for instance. Transferable Utility models can explain why individuals invest into their own education (beyond the benefits they reap on the labor market), or how an excess supply of marriageable women translates into higher welfare for husbands. It fails to explain how an increase in women's share of earnings could affect investment on children. These limitations explain why a more general class of models, based on Imperfectly Transferable Utility[4], has been developed. This is currently an area of active research.

The aim of the current survey is to provide an overview of these recent advances. We mostly concentrate on *bipartite, one-to-one* matching, e.g. on the traditional situation of marriage between one man and one

[1] See Chiappori and Salanié (2016) for a more detailed discussion of this point.

[2] Section 1.3 has a more prcise definition.

[3] As we shall see, under TU all efficient allocations maximize the unweighted sum of individual utilities.

[4] See Section 1.4.1.

woman. Sections 1.1 to 1.3 present the core of the theory. Similar tools can be applied to analyze same-sex marriages or polygamy[5]. A specificity of this survey is that we discuss dynamic aspects of mating markets at length[6]. Not only does Section 1.5 discuss divorce and remarriage; it also stresses the role of limited commitment, and the importance of premarital investments in shaping the marriage relationship.

Finally, we devote almost half of this chapter to empirical aspects. Section 2 describes the new methods that recent research has developed to identify and estimate models of mating markets. We focus on "separable" models with perfectly transferable utility, whose first instance was introduced by Choo and Siow (2006). We conclude by describing in Section 3 a few of the many recent empirical applications of the set of analytical tools presented in this survey.

While this chapter is limited to microeconomic approaches, marriage markets also have important implications for macroeconomic analysis; they are extensively discussed in Chapter 8 of this Volume.

1 Matching Markets: Theory

1.1 The Marital Surplus

From a theoretical perspective, the analysis of marriage relies on the simple but fundamental intuition that marriage generates a *surplus*: when married, two individuals can both achieve a higher level of well-being than they would as singles. The exact nature of the surplus is complex; depending on the issues under consideration, it may be described in different ways. Non-monetary aspects, including what is usually called love, certainly play an important role. Economists are often quite reluctant to model them in any specific manner; most of the time, these features are summarized by some random variable that represents, in a parsimonious way, the "quality" of the match. Economic benefits, on the other hand, are generated in ways that are more familiar to economists—from the existence of commodities that are publicly consumed within the family (children's welfare being a crucial example) to gender specialization to risk sharing. These aspects are discussed in detail in Chapter 3 of this Handbook; for the sake of completeness, we briefly survey them in this subsection.

[5] Same-sex marriage will be considered in Sections 1.4.3 and 3.6. The reader is referred to Azevedo and Hatfield (2018) for many to one matching, and to Reynoso (2019), André and Dupraz (2019) and Tapsoba (2021) for models of polygamy based on a matching approach.

[6] This is one of the main differences with other surveys, such as Chiappori (2020).

1.1.1 Consumption technology and domestic production

Public goods A first gain generated by marriage (or cohabitation) stems from the existence of commodities that are publicly consumed within the household. The cost of providing such commodities is split between members, which generates economic gains. These can be illustrated by a simple example in a two-person framework; extending the argument to larger households is straightforward. Consider a two-person household consuming two commodities, one private (individual consumptions being denoted q^A, q^B) and one public (common consumption Q); utilities are Cobb-Douglas

$$u^i\left(q^i, Q\right) = q^i Q \text{ for } i = A, B.$$

This example satisfies the Transferable Utility property that will be discussed in more detail in Section 1.3. Let x^A and x^B denote female and male income respectively, and let prices be normalized to 1. If single, spouses would each independently purchase (and privately consume) both commodities, leading to respective consumptions and utilities equal to

$$q^i = Q = \frac{x^i}{2} \text{ and } u_S^i = \frac{\left(x^i\right)^2}{4} \text{ for } X = A, B.$$

If the couple reaches an efficient decision, its aggregate consumption of the private good will satisfy

$$q^A + q^B = Q = \frac{x^A + x^B}{2},$$

resulting in utilities u_M^A and u_M^B that satisfy

$$u_M^A + u_M^B = \frac{\left(x^A + x^B\right)^2}{4}.$$

The marital surplus is simply:

$$S = \left(u_M^A + u_M^B\right) - \left(u_S^A + u_S^B\right) = \frac{\left(x^A + x^B\right)^2}{4} - \frac{\left(x^A\right)^2}{4} - \frac{\left(x^B\right)^2}{4} = \frac{x^A x^B}{2}$$

so that marriage has pushed up the utility possibilities frontier by $\left(x^A x^B\right)/2$ utils.

Economies of scale Alternatively, marital gains may coexist with purely private individual consumptions when the family is a source of economies of scale. This notion, which dates back (at least) to Becker (1981), has been abundantly investigated by the literature on "indifference scales" (see for instance Browning, Chiappori, and Lewbel, 2013).

Voena (2015) applies it to matching issues by assuming that spouses privately consume a single good, but that individual consumptions within a family may add up to more than the sum of individual consumptions of single individuals. In Voena's model, for instance, individual consumptions $\left(q^A, q^B\right)$ require total household expenditures equal to:

$$X = \left(\left(q^A\right)^\rho + \left(q^B\right)^\rho\right)^{1/\rho},$$

where the price of the unique good has been normalized to 1. For $\rho > 1$, one can readily check that $X < q^A + q^B$: the right hand side is the total cost faced by singles who would individually purchase the good.

Domestic Production and Specialization Margaret Reid (1934), then Gary Becker (1965; 1981) were among the first economists to stress that a large part of the total production of an economy takes place within households. Domestic production covers a large array of goods and services, from agricultural products to health care and food processing. Importantly, it also comprises investment in human capital—children's education being an obvious example.

Domestic production can easily be discussed using a variant of the previous model. Keeping the same utility functions as before, assume that the public good is produced from individual time, t^A and t^B respectively, according to the Cobb-Douglas production function:

$$Q = \left(0.1 + t^A\right)^{1/2} \left(0.1 + t^B\right)^{1/2}$$

Moreover, the time not devoted to children is spent on the labor market; let w_A and w_B denote individual wages, and let us normalize the total available time to 1.

Start with the behavior of a single parent, say A; we therefore assume that $t^B = 0$, and A's budget constraint is simply

$$q^A = w_A \left(1 - t^A\right)$$

Then A optimally chooses

$$q_S^A = \frac{2.2}{3} w_A \text{ and } t_S^A = \frac{0.8}{3}.$$

Considering now the household, aggregate budget constraint is:

$$q^A + q^B = w_A \left(1 - t^A\right) + w_B \left(1 - t^B\right)$$

and efficient allocations satisfy

$$t^A = \min\left(\frac{0.7}{4} + \frac{1.1 w_B}{4 w_A}, 1\right),\ t^B = \min\left(\frac{0.7}{4} + \frac{1.1 w_A}{4 w_B}, 1\right).$$

Individuals now specialize, as the time they each spend on domestic production depends the wage ratio w_B/w_A: the lower wage person spends more time on domestic production and less on salaried work. In particular, if the wage ratio is larger than 3 then $t^A = 1$: A leaves the labor market and exclusively specializes into the production of the public good. This specialization is a source of additional efficiency: the higher wage individual devotes more time to salaried work, while their spouse exploits their comparative advantage on domestic work.

This example calls for two remarks. If, following Becker (1981), we were to assume that individual times are perfect substitutes (i.e., production only depends on total time $\left(t^A + t^B\right)$), then efficiency typically would require full specialization ($\min\left(t^A, t^B\right) = 0$): the higher wage spouse does not spend any time on domestic production. In our example, time inputs are complements and the time input by each partner boosts the effectiveness of the other partner's investment. As a consequence, the high wage spouse always devotes some time (here 0.7/4 at least) to domestic production: specialization is only partial. Secondly, specialization would occur even if domestic production was consumed privately by each partner: efficiency always calls for time allocation to vary with wage rates and/or domestic productivities or preferences.

1.1.2 Risk Sharing

The household's ability to alleviate some market inefficiencies through bi- or multilateral agreements is another source of surplus. In the absence of complete insurance markets, individuals remain vulnerable to idiosyncratic shocks. Sharing the corresponding risk within the household potentially improves the (ex ante) welfare of all members. Assume for instance that household members consume a unique private good q^i ($i = A, B$), and individual VNM utilities are CARA:

$$u^i\left(q^i\right) = -\exp\left(-s^i q^i\right)/s^i$$

with $s^A, s^B > 0$ so that both partners are strictly risk averse. Each individual is endowed with a random income $\tilde{x}^i$. Once married, they can make ex ante efficient contracts, involving in particular risk sharing.

For any particular realization $x = \left(x^A, x^B\right)$ of individual incomes, let $(\rho^A(x), \rho^B(x))$ denote the individual consumptions. They are feasible if and only if

$$\rho^A(x) + \rho^B(x) = x^A + x^B. \tag{1}$$

We call a feasible pair $\left(\rho^A(x), \rho^B(x)\right)$ a *sharing rule*. If agents share risk efficiently, individual consumptions ρ^A and ρ^B only depend on total income $\bar{x} = x^A + x^B$:

Proposition 1 (Mutuality Principle) *If a sharing rule is efficient, then it only depends on the realization of total income:*

$$\rho^i(x) = \bar{\rho}^i\left(x^A + x^B\right) \quad i = A, B \tag{2}$$

for some functions $\bar{\rho}^i(\bar{x})$ *such that* $\bar{\rho}^A(\bar{x}) + \bar{\rho}^B(\bar{x}) \equiv \bar{x}$.
Proof. *Take any sharing rule* $\rho = \left(\rho^A, \rho^B\right)$ *and consider, for* $i = A, B$*:*

$$\bar{\rho}^i(\bar{x}) = \mathbb{E}\left[\rho^i\left(x^A, x^B\right) \mid x^A + x^B = \bar{x}\right].$$

$(\bar{\rho}^A(x^A + x^B), \bar{\rho}^B(x^A + x^B))$ *is clearly a sharing rule and for* $i = A, B$, *using Jensen's inequality:*

$$\begin{aligned}\mathbb{E}u^i\left(\bar{\rho}^i(\bar{x})\right) &= \mathbb{E}u^i\left[\mathbb{E}\left(\rho^i\left(x^A, x^B\right) \mid x^A + x^B = \bar{x}\right)\right] \\ &\geq \mathbb{E}\left[\mathbb{E}\left[u^i\left(\rho^i\left(x^A, x^B\right)\right) \mid x^A + x^B = \bar{x}\right]\right] \\ &= \mathbb{E}\left[u^i\left(\rho^i\left(x^A, x^B\right)\right)\right].\end{aligned}$$

Since both agents are strictly risk averse, Jensen's inequality is strict if ρ *has positive variance.This would violate efficiency. We cnclude that* $\rho = \bar{\rho}$ *a.s.:* $\rho^i(x^A, x^B)$ *only depends on* $\bar{x}$. ■

Under efficient risk sharing, each individual consumption only depends on the realization of household aggregate income, not on individual income shocks. Sharing total income, as opposed to individuals each bearing their idiosyncratic risk, creates an ex ante gain by allowing some degree of diversification. It is always beneficial, unless the two income streams are perfectly correlated and/or all agents are risk-neutral.

Efficiency also requires that the mappings $\left(\bar{\rho}^A(\bar{x}), \bar{\rho}^B(\bar{x}) = \bar{x} - \bar{\rho}^A(\bar{x})\right)$ maximize a weighted sum of individual expected utilities:

$$\bar{\rho}^A(\bar{x}) \in \arg\max_{r(\cdot)} \left(\mathbb{E}u^A\left(r(\bar{x})\right) + \mu\mathbb{E}u^B\left(\bar{x} - r(\bar{x})\right)\right)$$

for some $\mu > 0$. The first-order conditions give

$$\bar{\rho}^A(\bar{x}) = \frac{s^B\bar{x} - \ln\mu}{s^A + s^B} \quad \text{and } \bar{\rho}^B(\bar{x}) = \frac{s^A\bar{x} + \ln\mu}{s^A + s^B},$$

which results in individual expected utilities

$$\begin{aligned}\mathbb{E}u_M^A &= -\frac{\mu^{s^A/(s^A+s^B)}}{s^A}\mathbb{E}\left[\exp(-s\bar{x})\right] \\ \mathbb{E}u_M^B &= -\frac{\mu^{-s^B/(s^A+s^B)}}{s^B}\mathbb{E}\left[\exp(-s\bar{x})\right]\end{aligned} \tag{3}$$

where:

$$s = \frac{s^A s^B}{s^A + s^B} \Rightarrow \frac{1}{s} = \frac{1}{s^A} + \frac{1}{s^B}.$$

Here, s can be interpreted as the absolute risk-aversion of a *representative agent*. Indeed, assume that agents must choose between several possible income distributions $\left(\tilde{x}^A, \tilde{x}^B\right)$; their (unanimous) choice, irrespective of the particular Pareto weight μ, will select the distribution that maximizes the expression $\mathbb{E}\left[\exp(-s\bar{x})\right]$. This would also be the choice of a representative agent with the vNM utility $U(x) = -\exp(-sx)$; in fact, the household will, on aggregate, always behave in exactly the same way as the representative agent. As we shall see below, this property characterizes the Identical Shape Harmonic Absolute Risk Aversion family (ISHARA—which contains CARA utility functions).

Finally, expected utilities as single are $\mathbb{E}u^i_S = -\mathbb{E}\left[\exp(-s^i x^i)\right]/s^i$. Define the certainty equivalents C^A_S, C^B_S as single by $u^i(C^i_S) = Eu^i(x^i)$, and the certainty equivalent C of the representative agent by $\mathbb{E}\left[\exp(-s\bar{x})\right] = \exp(-sC)$. One can readily check that $C > C^A_S + C^B_S$, as predicted by the usual insurance argument. In other words, there exists an open interval of values of the Pareto weight μ that provide both agents with more utility than they would get as single.

Marital gains can be realized under other types of market imperfections. For instance, if individuals cannot borrow against their future income, singles may be unable to achieve socially efficient investments in human capital. A couple may be able to relax the corresponding liquidity constraint by having one spouse work while the other studies. This is quite similar to the risk-sharing framework. However, the individuals' ability to commit (which was taken for granted in the previous example) raises new and interesting issues which we will discuss in Section 1.5.

1.2 Mating Models: A Taxonomy

While formal models of mating markets differ in many aspects, they all share a common feature: they consider individuals who are fundamentally heterogeneous. Following the standard approach of the hedonic literature, this heterogeneity can be described by a list of characteristics (or "traits"). As a consequence, individuals typically have different valuations of the observable characteristics of potential mates.

The fundamentals of marriage markets consist of two components: a description of the two populations, and an evaluation of the benefits that would be generated by the match of *any two* potential spouses[7]. Any

[7] Throughout this survey, a "match" is the association of two specific individuals, and a "matching" is the collection of matches (or individual singlehood) over the entire population.

theoretical analysis of the market must answer two sets of questions:

Q1: the equilibrium matching patterns—who stays single, and who marries whom?

Q2: the equilibrium payoffs—how is the marital surplus distributed between the spouses?

These questions have been analyzed within two different frameworks: frictionless matching theory and search models. The basic distinction between the two is related to the role given to *frictions* in the description of the market.

1.2.1 Search and Frictionless Matching

In search models, frictions are paramount. Typically, individuals each sequentially and randomly meet one person of the opposite gender; after such a meeting, they both must decide whether to settle for the current mate or to continue searching. The latter option involves various costs, from discounting to the risk of never finding a better partner. If both individuals agree to engage in a relationship, then a negotiation begins on how the surplus is to be shared.

Matching models, on the contrary, assume a frictionless environment. In the matching process, each individual is assumed to have free access to the pool of all potential spouses, with perfect knowledge of the characteristics of each of them. Matching models thus disregard the cost of acquiring information about potential matches, as well as the role of meeting technologies of all sorts (from social media to dating sites to pure luck).

1.2.2 Utility Transfers

Within the family of frictionless matching frameworks, a second and crucial distinction relies on the role of transfers: are partners in a match able to transfer utility to each other? Transfers make a fundamental difference: when available, they allow agents to "bid" for their preferred mate by offering to reduce their own gain from the match in order to increase the partner's. The nature of these bids depends on the context; they need not take the form of monetary transfers. In family economics, they may affect the allocation of time between paid work, domestic work and leisure; the choice between current and future consumption; or the structure of expenditures for private or public goods. Whatever form they take, utility transfers enable agents to negotiate, compromise, and ultimately exploit mutually beneficial solutions.

The literature on matching has mostly focused on two polar extremes. In the so-called Non Transferable Utility (NTU) case, there is simply no technology enabling agents to transfer utility to any potential partner[8]. This framework has been applied successfully to a host of important issues, from the allocation of residents to hospitals (Roth, 1984), to kidney exchange (Roth, Sönmez, and Ünver, 2005) or the allocation of students to public schools (Abdulkadiroglu and Sönmez, 2003).

In the case of marriage, however, the NTU case appears to be much less relevant. If at least one commodity is privately consumed by family members, then different efficient allocations typically correspond to different individual private consumptions; choosing one of these allocations against the others is formally equivalent to a transfer. Even if all consumptions were public, members would typically disagree on the household's preferred bundle; again, compromises along that dimension amount to transfers between spouses. In fact, it is hard to imagine situations where any move along the utility possibility frontier of the partners is simply ruled out. In this survey, we shall thus concentrate on matching models involving transfers[9].

When transfers are possible, the surplus created by a match must be allocated between partners. An equilibrium must therefore specify not only matching patterns—who is matched with whom—but also the supporting division of the surplus; the latter is now *endogenous* and determined (or at least constrained) by equilibrium conditions on the marriage market[10]. The answers to both questions **Q1** and **Q2** are inextricably linked in this framework.

We will focus on the extreme opposite of NTU, in which transfers are costless, unlimited, and have the same constant marginal value for all individuals. Then the Pareto frontier, which represents the set of utility pairs that are just feasible given resource constraints, is a straight line with slope -1 for all values of prices and incomes. That is, for a well-chosen cardinalization of individual utilities, increasing a partner's utility by one util has a cost of exactly one util, irrespective of the economic environment (prices, incomes, ...) In this setting of perfectly Transferable Utility, which we will denote TU hereafter, any given match generates a total gain that is *additively* split between the two partners.

[8]The interested reader is referred to Roth and Sotomayor's excellent monograph (Roth and Sotomayor, 1990).

[9]Non transferable utility models have been applied to dating; see Hitsch, Hortacsu, and Ariely (2010) and Banerjee, Duflo, Ghatak, and Lafortune (2013). In societies ruled by very rigid social norms, transfers may be dictated by custom rather than be determined endogenously in equilibrium. As we will discuss in Section 1.5, transfers may also be problematic in the absence of (any form of) intertemporal commitment.

[10]This is exactly Becker's original intuition: "[...] theory does not take the division of output between mates as given, but rather derives it from the nature of the marriage market equilibrium". (Becker (1973), p. 813).

A more general version (often called ITU for Imperfectly Transferable Utility) allows for transfers, but recognizes that the "exchange rate" between individual utilities is not constant, and typically endogenous to the economic environment. We will return to it in Section 1.4.1.

1.3 Matching Models under Transferable Utility

1.3.1 The Basic Framework

Let us start with some notation. We consider two compact sets $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{Y} \subset \mathbb{R}^m$, which respectively represent the space of female and male characteristics. The corresponding vectors of characteristics fully describe the agents; i.e., for any $x \in \mathcal{X}$, two women with the same vector of characteristics x are perfect substitutes as far as matching is concerned (and similarly for men). These spaces are endowed with measures F and G respectively; both $F(\mathcal{X})$ and $G(\mathcal{Y})$ are finite. In order to capture the case of persons remaining single within this framework, a standard trick is to "augment" the spaces by including an isolated point in each: a dummy partner $\emptyset_X$ for any unmatched man and a dummy partner $\emptyset_Y$ for any unmatched woman. Therefore, from now on we consider the spaces $X := \mathcal{X} \cup \{\emptyset_X\}$ and $Y := \mathcal{Y} \cup \{\emptyset_Y\}$, where the point $\emptyset_X$ (resp. $\emptyset_Y$) is endowed with a mass measure equal to the total measure of $\mathcal{Y}$ ($\mathcal{X}$). In particular, a hypothetical matching in which all women remain single would be described by matching them all with $\emptyset_Y$.

As we explained in Section 1.2.2, with transferable utility the answers to questions **Q1** and **Q2** are linked. To answer question **Q1** ("Who marries whom?"), we define a measure h on $X \times Y$; intuitively, one can think of $h(x, y)$ as the probability that x is matched to y in the matching h. Note that this definition allows for *randomization*. Randomization simplifies the problem by convexifying it; moreover, allowing for randomization is sometimes necessary.[11] When each x has a unique match $y = \phi(x)$, and conversely, the matching is said to be *pure*; it will be the case at equilibrium in many of the examples considered in this chapter.

A matching h is *feasible* if its *marginals* on X and Y are F and G respectively; formally:

Definition 2 (Feasible Matching) *A measure h on $X \times Y$ is a feasible matching if and only if for all $x \in X$ and $y \in Y$,*

$$\int_{t \in Y} dh(x, t) = F(x) \text{ and } \int_{z \in X} dh(z, y) = G(y). \qquad (4)$$

[11] See Chiappori, McCann, and Nesheim (2010) for examples in which the *unique* equilibrium matching requires randomization for an open subset of characteristics.

Note that the feasibility constraints are *linear* in h, a point that will become important later on.

The (perfectly) TU case relies on the additional assumption that, for a well chosen cardinalization of individual utilities, a potential match between x and y generates a *joint surplus* $S(x,y)$ that is *additively* split into the individual surpluses of the two partners. The joint surplus, which corresponds to the marital surplus of Section 1.1, is then the differences between the sum of utilities that the spouses can reach when matched and the sum of their individual utilities if both stay single. In particular, the "surplus" generated by singlehood (i.e., a match with the dummy partner $\emptyset_X$ or $\emptyset_Y$) is zero. This brings us to question **Q2**: how is the surplus split?

Consider any feasible matching h. If x and y are matched with positive probability under h, we denote $u(x)$ and $v(y)$ their individual surpluses if they match, and we have:

$$h(x,y) > 0 \Rightarrow u(x) + v(y) = S(x,y). \tag{5}$$

Condition (5) simply states that matched people share the resulting surplus. Note that if x stays single, then $u(x) = S(x, \emptyset_Y) = 0$.

Like most of the literature, we model equilibrium by assuming *stability* (Gale and Shapley, 1962; Shapley and Shubik, 1972).

Definition 3 (Stable Matchings) *A matching is stable iff it is feasible and:*

(i) no matched individual would prefer being single, and

(ii) no pair of individuals would both *prefer being matched together (for a well-chosen distribution of the surplus) over their current situation.*

Requirement (ii) implicitly incorporates a notion of "divorce at will": whenever it is violated by a pair of individuals, if (s)he is currently matched she will divorce their current spouse *at no cost* to form a new union.

One can readily see that stability requires the following inequalities:

$$u(x) + v(y) \geq S(x,y) \quad \forall (x,y) \in X \times Y \tag{6}$$

Indeed, assume there exists a pair $(x,y) \in X \times Y$ such that $u(x)+v(y) < S(x,y)$. Then by (5), x and y are matched with zero probability; yet they could both strictly benefit from being matched together, since the surplus $S(x,y)$ they generate is sufficient to provide x with strictly more than $u(x)$ and y with strictly more than $v(y)$. But that would violate requirement (ii) of stability.

An equivalent statement is the following: if a matching h is stable, the corresponding functions u and v, from X to $\mathbb{R}$ and from Y to $\mathbb{R}$ respectively, are such that:

$$u(x) = \max_{t \in Y} \{S(x,t) - v(t)\} \quad (7)$$

$$\text{and } v(y) = \max_{z \in X} \{S(z,y) - u(z)\}; \quad (8)$$

and in each of these equalities, the maximum is reached for all potential spouses (possibly including the dummy one) to whom the individual is matched with positive probability under h. Note that (7) has a natural interpretation in hedonic terms: $v(y)$ is the "price" (in utility terms) that x would have to pay should she choose to marry y; then she would keep what is left of the surplus, namely $S(x,y) - v(y)$. Obviously, the same argument applies (*mutatis mutandis*) to (8).

1.3.2 Household behavior and TU

In the context of a family, the TU property states that, for well chosen cardinalizations of individual preferences, the Pareto frontier generated by a given budget constraint is a straight line with slope -1 *for all values of prices and incomes*. That is, its equation is simply:

$$u^A + u^B = \Phi \quad (9)$$

for some function Φ of prices and income[12]. This, in turn, requires specific assumptions on individual preferences, that we now describe.

We consider a two-person (A,B) household (the extension to any number of individuals is straightforward). The household consumes n private goods and N public goods; an allocation thus is a $(2n+N)$-vector

$$\mathcal{Q} = \left(q_1^A, \ldots, q_n^A, q_1^B, \ldots, q_n^B, Q_1, \ldots, Q_N\right)$$

We assume egoistic preferences of the form $u^i(q^i, Q)$ for $i = A, B$, and we define the *conditional indirect utility* of i by:

$$v^i(p, Q, \rho) = \max_q \left\{u^i(q, Q) \mid p'q = \rho\right\}.$$

In words, $v^i(p, Q, \rho)$ is the maximum utility that individual i can reach when consuming the vector Q and optimally choosing their private consumption subject to the budget constraint $p'q = \rho$.

[12] For a k-person household, TU requires that the Pareto frontier can, for adequate cardinalizations, be represented as an hyperplane orthogonal to the unit vector, with equation $\sum_k u^k = \Phi$.

A Basic Model As is well known (see for instance Browning, Chiappori, and Weiss (2014)), any efficient allocation can be interpreted as the outcome of a two-stage decision process. In stage 1, members collectively choose the household demands for public goods Q and decide how the remaining income $x - P'Q$ is split between members. We denote ρ^i the income of member $i = A, B$, with $\rho^A + \rho^B = x - P'Q$. In stage 2, each member independently decides on their private consumption q^i under the budget constraint $p'q^i = \rho^i$, and achieves conditional indirect utility $v^i(p, Q, \rho^i)$. As a consequence, any efficient first stage choice solves:

$$\max_{Q,\rho^A,\rho^B} v^A\left(p, Q, \rho^A\right) + \mu v^B\left(p, Q, \rho^B\right)$$

under the constraint

$$\rho^A + \rho^B = x - P'Q$$

for some scalar $\mu > 0$.

Chiappori and Gugl (2020) proved that TU holds for a pair of preferences if and only if they can be represented by conditional indirect utility functions that are affine in private expenditures and share the same slope.

Definition 4 (ACIU) *A utility function u^i satisfies the Affine Conditional Indirect Utility (ACIU) property if one can find a continuous scalar function $\alpha^i(Q, p)$ from $\mathbb{R}^{N+n}$ to $\mathbb{R}$ that is (-1)-homogeneous in p, and a continuous scalar function $\beta^i(Q, p)$ from $\mathbb{R}^{N+n}$ to $\mathbb{R}$ that is 0-homogeneous in p, such that the conditional indirect utility corresponding to u^i can be written as:*

$$v^i(p, Q, \rho) = \alpha^i(p, Q)\,\rho + \beta^i(p, Q) \quad \text{for all } (p, Q, \rho)\,. \tag{10}$$

Proposition 5 (Characterization of TU Preferences) *A pair of preferences satisfy the TU property if and only if one can find two representations $\left(u^A, u^B\right)$ that both satisfy the ACIU property (10), with moreover*

$$\alpha^A(p, Q) = \alpha^B(p, Q)\,. \tag{11}$$

Proof. *See Chiappori and Gugl (2020).* ■

The property defined in Proposition 5, which can be called ISACIU (for Identical Shape Affine Conditional Indirect Utility), is thus necessary and sufficient for utility to be perfectly transferable. Chiappori and Gugl (2020) list some of the functional forms that satisfy this property.

Note also that under TU, the household behaves as a single individual who would maximize the sum of individual utilities; in particular, the household's demand for public goods is the same for all Pareto efficient allocations[13].

Uncertainty: the one-dimensional case The TU property can be characterized in more complex frameworks. Here we provide a result for the case of decision under uncertainty; we will discuss the transposition to an intertemporal model in Section 1.5.

Let us start with the model of Section 1.1.2, where two agents $i = A, B$ consume a numéraire good. Given a feasible sharing rule (ρ^A, ρ^B), the expected utility of agent i is $\mathbb{E}v^i\left(\rho^i\left(\tilde{x}^A, \tilde{x}^B\right)\right)$, where v^i is i's (indirect) von Neumann-Morgenstern utility and the expectation is taken over the distribution of $\left(\tilde{x}^A, \tilde{x}^B\right)$. As always, ex ante efficiency requires that no alternative sharing rule could increase expected utility for both individuals. By the mutuality principle (Proposition 1), the efficiency sharing rule only depends on total income $\bar{x} = \tilde{x}^A + \tilde{x}^B$: for $i = A, B$, it is of the form $\rho^i(\bar{x})$.

Mazzocco (2004) and Schulhofer-Wohl (2006) provide a characterization of vNM utilities that exhibit the TU property. As before, we start with a definition:

Definition 6 *A pair of vNM utility functions $\left(v^A, v^B\right)$ belongs to the Identical Shape Harmonic Absolute Risk Aversion (ISHARA) class if the corresponding indices of absolute risk aversion are harmonic:*

$$-\frac{d^2v^i/\partial\rho^2}{dv^i/\partial\rho} = \frac{1}{a^i + b^i\rho} \tag{12}$$

for $i = A, B$, and moreover $b^A = b^B$.

Condition (12) expresses that for each individual utility, the index of Absolute Risk Aversion is an Harmonic function of income; the shape coefficient is then b^i, and the Identical Shape requirement imposes $b^A = b^B$. For instance, any pair of CARA utility functions always belong to the ISHARA class (with $b^A = b^B = 0$), whereas two CRRA utilities are ISHARA if and only if they have the same coefficient of relative risk aversion b (then $a^A = a^B = 0$ and $b^A = b^B = b$).

Proposition 7 (ISHARA implies TU) *Consider a pair of vNM utility function $\left(v^A, v^B\right)$ that belongs to the ISHARA class, and assume that individuals share their income risk efficiently. Then:*

[13] The converse is false; one can easily generate examples in which the household behaves as a single individual (in particular, its demand for public good is identical for all efficient allocations) but that fail to satisfy the TU property.

1. *The sharing rule is an affine function of household income.*

2. *The household behaves as a single consumer, in the sense that all efficient sharing rules generate the same aggregate behavior; the latter maximizes expected utility for some representative vNM utility U that is also HARA.*

3. *The model is TU, in the sense that there exists two increasing mappings f^A, f^B from $\mathbb{R}$ to $\mathbb{R}$ such that, for any probability distribution of $\left(\tilde{x}^A, \tilde{x}^B\right)$, all efficient sharing rules $\left(\rho^A, \rho^B\right)$ satisfy*

$$f^A\left(\mathbb{E}v^A\left(\rho^A(\bar{x})\right)\right) + f^B\left(\mathbb{E}v^B\left(\rho^B\left(\bar{x}\right)\right)\right) = K \tag{13}$$

   where K depends on the preferences and on the distribution of the total income $\bar{x}$.

4. *In particular, if $\left(\rho^A, \rho^B\right)$ is an efficient sharing rule, let C_M^i denote the certainty equivalent, for i, of the (random) allocation $\rho^i\left(\tilde{x}\right)$. Then*

$$C_M^A + C_M^B = C$$

   where C is the certainty equivalent, for the representative consumer, of the random allocation $\tilde{x}$; in particular, C does not depend on the choice of the efficient sharing rule.

Conversely, if the four previous properties are satisfied for all probability distributions $\left(\tilde{x}^A, \tilde{x}^B\right)$, then the pair of vNM utility function $\left(v^A, v^B\right)$ belongs to the ISHARA class.

Proof. *See Mazzocco (2004) and Schulhofer-Wohl (2006).* ■

Condition (13) is a particular case of the general TU requirement (9). In words, the ex ante welfare of individual i can be measured by i's expected utility $\mathbb{E}v^i$, but also, equivalently, by any increasing function of $\mathbb{E}v^i$. In particular, i's ex ante welfare can be measured by i's certainty equivalent C_M^i. Then individual certainty equivalents *add up* to the same certainty equivalent C for all ex ante efficient allocations; moreover, C can be interpreted as the certainty equivalent of the representative consumer.

As an illustration, consider the CARA utility functions of Section 1.1.2. We had

$$\begin{aligned} -\log\left(-\mathbb{E}u_M^A\right) &= \log s^A - \frac{s^A}{s^A+s^B}\log\mu - \log\mathbb{E}\exp(-s\bar{x}) \\ -\log\left(-\mathbb{E}u_M^B\right) &= \log s^B + \frac{s^B}{s^A+s^B}\log\mu - \log\mathbb{E}\exp(-s\bar{x}). \end{aligned}$$

This directly implies

$$-\frac{1}{s^A}\log\left(-\mathbb{E}u_M^A\right)-\frac{1}{s^B}\log\left(-\mathbb{E}u_M^B\right)=\frac{\log s^A}{s^A}+\frac{\log s^B}{s^B}-\frac{1}{s}\log\mathbb{E}\exp(-s\bar{x})$$

which is of the form (13) for $f^i(t) = -\log(-t)/s^i$. In certainty equivalent terms:

$$C^A = L - \ln\left(\int \exp\left(-\frac{s^A s^B}{s^A+s^B}x\right)dF(x)\right) \quad \text{and}$$
$$C^B = -L - \ln\left(\int \exp\left(-\frac{s^A s^B}{s^A+s^B}x\right)dF(x)\right)$$

where the constant L depends on the Pareto weight μ. For all values of μ, we have:

$$C^A + C^B = -\ln\left(\int \exp\left(-\frac{s^A s^B}{s^A+s^B}x\right)dF(x)\right).$$

Note that this is the certainty equivalent of a representative consumer whose risk tolerance is the sum of the risk tolerances of A and B:

$$\frac{1}{s}=\frac{1}{s^A}+\frac{1}{s^B}.$$

Uncertainty: the general case The previous result can readily be extended to the multiple-goods framework. Specifically, assume that (i) individual preferences satisfy the ISACIU property for some well-chosen cardinalization, and (ii) individual vNM utilities, considered as functions of individual private incomes, belong to the ISHARA class. Then the model is TU.

To see why, start with the ISACIU property: for $i = A, B$,

$$v^i(p,Q,\rho) = \alpha(p,Q)\rho + \beta^i(p,Q). \tag{14}$$

Since ex ante efficient allocations are also ex post efficient, for any income realization the choice of the public consumption vector Q must maximize the sum of utilities *using the cardinalization corresponding to the ACIU property*. That is, Q solves:

$$\max_Q\left(\alpha(p,Q)(\bar{x}-P'Q) + \sum_i \beta^i(p,Q)\right).$$

Let $\bar{Q}$ denote the solution; note that $\bar{Q}$ only depends on prices and on total household income $\bar{x}$. Now assume that the vNM utility of $i = A, B$

is $\phi^i\left(v^i\left(p, Q, \rho\right)\right)$, where the pair $\left(\phi^A, \phi^B\right)$ belongs to the ISHARA class. Any ex ante efficient allocation must solve, for some $\mu > 0$,

$$\max_{\rho^A, \rho^B} \mathbb{E}\left[\phi^A\left(v^A\left(p, \bar{Q}, \rho^A\right)\right)\right] + \mu \mathbb{E}\left[\phi^B\left(v^B\left(p, \bar{Q}, \rho^B\right)\right)\right].$$

Given the ISACIU property, this can be rewritten as

$$\max_{W^A, W^B} \mathbb{E}\left(\phi^A\left(W^A\right) + \mu\phi^B\left(W^B\right)\right),$$

where $W^i = v^i\left(p, \bar{Q}, \rho^i\right)$, under the constraint that

$$W^A + W^B = \alpha\left(p, \bar{Q}\right)\left(\bar{x} - P'\bar{Q}\right) + \beta^A\left(p, \bar{Q}\right) + \beta^B\left(p, \bar{Q}\right) \equiv \bar{W}.$$

By Proposition 7, there exist $\left(f^A, f^B\right)$ such that all ex ante efficient allocations solve:

$$f^A\left(\mathbb{E}\left[\phi^A\left(W^A\right)\right]\right) + f^B\left(\mathbb{E}\left[\phi^B\left(W^B\right)\right]\right) = K$$

which is exactly TU.

1.3.3 Duality and Supermodularity

Optimal transportation and duality A crucial property of matching models under TU is their intrinsic relationship with a class of linear maximization problems called "optimal transportation"[14] Consider the following question: Find a measure h on $X \times Y$, the *marginals* of which are F and G respectively, that maximizes the integral

$$\mathcal{S} = \int_{X\times Y} S\left(x, y\right) dh\left(x, y\right). \tag{15}$$

From an economic perspective, this problem has a straightforward interpretation; just think of a benevolent dictator who can match people at will, and is trying to maximize total welfare. In a TU framework, where individual utilities can all be measured in the same units, the natural measure of total welfare is the sum of all surpluses generated; that is exactly the meaning of the right-hand side integral in (15).

As this problem is linear in h, its value coincides with that of its dual[15]. The dual problem consists in finding two functions u and v, respectively defined on X and Y, that minimize the sum

[14] The problem was introduced by Gaspard Monge in 1781 for military engineering; Kantorovitch applied linear programming techniques to it in the 1940s.

[15] See Galichon (2016) or Chiappori (2017) for a more detailed exposition of duality theory.

$$\tilde{\mathcal{S}} = \int_i u(x)dF(x) + \int_Y v(y)dG(y) \tag{16}$$

under the constraints:

$$u(x) + v(y) \geq S(x, y) \quad \forall (x, y) \in X \times Y$$

Note that these constraints are simply the stability constraints of (6).

Under mild conditions, if h is a solution to the primal problem, then it is the measure of a stable matching, and any associated equilibrium utilities u and v solve the dual problem. Reciprocally, if u and v solve the dual problem, then any solution h to the primal problem has its support in the set of (x, y) for which $u(x) + v(y) = S(x, y)$.

To summarize: finding a stable matching boils down to the resolution of a linear optimization problem. Since the constraints obviously define a non-empty feasible set, a stable matching obtains under mild continuity and compactness conditions. The corresponding measure h is generically unique—in the sense that while examples with multiple stable matchings can be constructed, they are not robust to small perturbations.

Supermodularity The one-dimensional case $m = n = 1$ allows us to introduce an important notion: the supermodularity of the surplus.

Definition 8 (Supermodularity) *A function $f : \mathbb{R}^2 \to \mathbb{R}$ is supermodular if and only if for for all $x \leq x'$ and $y \leq y'$,*

$$f(x, y) + f(x', y') \geq f(x, y') + f(x', y) \tag{17}$$

If f is twice continuously differentiable, supermodularity is equivalent to the Spence-Mirrlees condition:

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) \geq 0 \quad \forall x, y.$$

When the surplus S is strictly supermodular, the only stable matching must be *positively assortative*; for any two matched couples (x, y) and (x', y'), if $x < x'$ then $y \leq y'$. With continuous distributions, matching patterns follow a simple rule: x is matched to y if and only if the total mass of matched women above x equals the total mass of matched men above y, that is (assuming equal total numbers of men and women) $1 - F(x) = 1 - G(y)$. Formally, the matching is pure and can be described by a function

$$y = (G^{-1} \circ F)(x).$$

In particular, all supermodular surplus functions generate exactly the same matching patterns.

Lastly, if (17) holds with the opposite inequality, then the surplus function is *submodular*, and the stable matching is now negative assortative (larger x match with smaller y and conversely).

1.3.4 Multidimensional matching under TU

The previous approach can be extended to multi-dimensional settings.

Index Models In the so-called index model, the various characteristics of at least one partner only enter the surplus through some one-dimensional index[16]:

$$S\left(x_1, \ldots, x_n, y\right) = \overline{S}\left(I\left(x_1, \ldots, x_n\right), y\right) \tag{18}$$

for some functions $\overline{S}$ and I. The index I serves as an aggregator of the vector of characteristics $x = (x_1, \ldots, x_n)$ that fully reflects her "attractiveness" on the marriage market: two women with different vectors x, x' but the same index value ($I(x) = I(x')$) are perfect substitutes.

Suppose for simplicity that all characteristics are continuous. In a multidimensional setting, there exist trade-offs between the various traits that characterize a woman. They are described by the ratio (formally equivalent to a marginal rate of substitution)

$$M_{ij}\left(x, y\right) = \frac{\partial S / \partial x_j}{\partial S / \partial x_i}(x, y)$$

In words, M_{ij} represents the infinitesimal amount by which the j-th trait must be increased to compensate for an infinitesimal reduction in the i-th and leave the marital surplus unchanged. In general, M_{ij} is y-specific: two different men would disagree on the value of the ratio. An index framework, on the contrary, postulates that the compensation is evaluated in exactly the same way by all men: $M_{ij}(x, y)$ is the marginal rate of substitution of the index I at x, therefore does not depend on y.

Index models have very specific properties. In particular, stable matchings are not unique any more. Indeed, all individuals with the same index value are perfect substitutes, and can therefore be arbitrarily reshuffled between matches. In particular, a double index model $S(x, y) = \bar{S}(I(x), J(y))$ is essentially one-dimensional: if for instance $\overline{S}$ is supermodular, index values must be matched assortatively. On the other hand, individual matching conditional on the index values is indeterminate.

Many-to-one dimensional matching Another interesting situation obtains when dimensions m and n differ. Assume for instance that

[16] This can easily be extended to a multi-index model.

$m = 1$ but $n \geq 2$. Then a husband with a given characteristic y will marry with positive probability any of a continuum of different women x, thus defining "iso-husband" curves in the space of female characteristics. Note that these curves are (in principle) identifiable from data on matching patterns.

Theory generates testable predictions relating the surplus function to the shape of iso-husband curves. To see how, let us consider the case $n = 2$. The stability condition:

$$v(y) = \max_{x_1, x_2 \in X} \{S(x_1, x_2, y) - u(x_1, x_2)\}$$

gives by the envelope theorem:

$$v'(y) = \frac{\partial S}{\partial y}(x_1, x_2, y) \tag{19}$$

which defines an iso-husband curve.

In the case of an index model $S(x_1, x_2, y) = \bar{S}(I(x_1, x_2), y)$ the equation boils down to $I(x_1, x_2) = K(y)$, where K is a function that depends on the marginal distributions of (x_1, x_2) and y. Then the set of iso-husband curves is simply the set of iso-index curves, and the model boils down to a one-dimensional matching between y and the index $I(x)$.

In a general (non index) framework, however, the *shape* of the iso-husband curves also depend on the marginals. Equation (19) still yields the following:

Proposition 9 *Assume that the cross-derivatives* $\frac{\partial^2 S}{\partial x_1 \partial y}$ *and* $\frac{\partial^2 S}{\partial x_2 \partial y}$ *are positive. Then the iso-husband curves are decreasing in the* (x_1, x_2) *plane.*

Proof. From the implicit function theorem, (19) implies that the equation of the iso-husband curve can be written as:

$$x_2 = \phi(x_1, y)$$

with

$$\frac{\partial \phi}{\partial x_2}(x_1, y) = -\frac{\frac{\partial^2 S}{\partial x_2 \partial y}}{\frac{\partial^2 S}{\partial x_1 \partial y}}(x_1, x_2, y) < 0.$$

■

This result can be seen as an extension of the supermodularity property discussed in the one-dimensional case. Note that the sign of the cross derivative $\partial^2 S(x_1, x_2, y) / \partial x_1 \partial x_2$ is irrelevant to this result, as is the distribution of characteristics.

Finally, an intuitive property of the stable matching would be the following: if $\frac{\partial^2 S}{\partial x_1 \partial y} > 0$ and $\frac{\partial^2 S}{\partial x_2 \partial y} \geq 0$, then for any woman (x_1, x_2) on a given iso-husband curve corresponding to some husband y, all women located *above* that curve (in the (x_1, x_2) plane) are matched with a husband with a characteristic y' larger than y. This property, which constitutes a direct extension of supermodularity, holds true for index models. In the general case, things may be more complex, as curves defined by (19) for different values of y may intersect. One then has to check a regularity property called *nestedness*. The interested reader is referred to Chiappori, McCann, and Pass (2017).

1.4 Extended Matching Models

1.4.1 Matching under Imperfectly Transferable Utility (ITU)

TU models rely on a very specific property: for a well-chosen cardinalization of individual preferences, the Pareto frontier is a hyperplane orthogonal to the unitary vector *for all price and incomes.* A more general utility possibility set can be defined by an equation of the form:

$$U \leq \Phi(x, y, V) \tag{20}$$

where U and V are the utilities of the partners and Φ is non-increasing in V. The TU case corresponds to a quasi-additive $\Phi(x, y, V) = S(x, y) - V$; with NTU , the utility possibility set consists on one point only ($U = U(x, y), V = V(x, y)$).

A matching is still defined as a 3-uple (h, u, v) where the marginals of measure h are F and G respectively, and (20) is satisfied with equality whenever $h(x, y) > 0$. Stability requires, moreover, that:

$$u(x) \geq \Phi(x, y, v(y)) \quad \forall x, y \tag{21}$$

with the same interpretation as in the TU case. In particular, $u(x)$ must be the value of the maximum over y of $\Phi(x, y, v(y))$, so that at the stable matching

$$u'(x) = \frac{\partial \Phi}{\partial x}(x, y, v(y)).$$

Since the notion of total surplus is not defined in this framework, a fortiori stability is no longer equivalent to surplus maximization. Moreover, supermodularity (in the sense that $\partial^2 \Phi / \partial x \partial y > 0$) is no longer sufficient to guarantee assortative matching; the conditions also involve the cross-derivative $\partial^2 \Phi / \partial x \partial v$. Intuitively, $\partial \Phi / \partial v$ represents the "exchange rate" between his and her utility; unlike in the TU case, this rate is not constant, and the sign of $\partial^2 \Phi / \partial x \partial v$ indicates how it changes with the wife's characteristics[17].

[17] See Legros and Newman (2007) or Chiappori (2017).

ITU models provide a generalization that overcomes a well-known weakness of the TU setting: under TU, the household behaves as a single decision maker whose "preferences" are independent of the economic context. Indeed, the TU assumption implies that, for a well chosen cardinalization of individual preferences, all efficient allocations maximize the sum of individual utilities. Changes in the environment, in the distributions of individual characteristics in particular, may affect the balance of power within the household (and therefore individual consumptions). However, they cannot possibly impact its aggregate behavior, i.e. the total household demand for either private or public goods. For instance, an increase in female income or education may empower women, but only in a very specific sense: the share of private goods that goes to the wife will increase, but the total expenditures on children or the budget share of housing will not.

In an ITU context, on the contrary, different efficient allocations correspond to different patterns of household behavior; as a result, changes in the economic environment that shift the intrahousehold allocation of decision power directly trigger aggregate behavioral responses. Female empowerment may for instance result in more resources being allocated to children.

This generalization, however, comes at a cost. In particular, the equivalence between stability and the maximization of total surplus is lost as the mere notion of a "joint surplus" is no longer defined. The existence and efficiency of a stable matching can generally still be established[18]. However, generic uniqueness no longer obtains. Moreover, the empirical estimation of ITU models raises specific difficulties, due in particular to the non-linearities and the additional parameters introduced by the ITU framework; see Galichon, Kominers, and Weber (2019) for recent advances on this topic.

1.4.2 Search models

Search models play a crucial role in labor economics; modern approaches recognize that unemployment is due, at least in part, to search frictions on the labor market. Two of the early and seminal papers in the search literature, Mortensen (1982, 1988), explicitly referred to the marriage market as a prime application of search models. Later attention focused on the labor market[19]. Still, there has been a revival of interest in search in marriage since Shimer and Smith (2000). The basic framework is generally similar to the one developed above. Each women (resp. man)

[18] See for instance Legros and Newman (2007) or Chiappori and Reny (2016), and Greinecker and Kah (2019) for a general result.

[19] See for instance Pissarides (2000).

has a vector of characteristics $x \in X$ (resp. $y \in Y$), and a match between x and y generates a marital surplus $s(x, y)$ that must be shared between spouses in a TU framework. The new element is the recognition that men and women must meet before they can match, and that such meetings take time. This search friction introduces a trade-off between matching now or deciding to wait for the chance of meeting another potential partner and achieving higher surplus. Waiting has a cost, both because of discounting and because a better partner may never show up.

In the standard version, meetings between men and women occur randomly; this is typically modeled as a Poisson process. In addition, new individuals enter the market as singles; and some matches are dissolved. When a meeting takes place, the partners bargain over the division of the surplus, given their perceptions of market opportunities and of the cost of waiting. With search frictions, if a match occurs it must generate a higher surplus than the alternative of waiting. This puts the two partners in a situation of bilateral monopoly, and the model must include assumptions on the bargaining process used to allocate welfare within the couple. One must also assume that partners commit to the outcome of the bargaining process. Most of the literature has concentrated on the steady state[20].

Let us illustrate these ideas with the Shimer and Smith (2000) model. They consider a continuum of infinitely-lived men and women, with one-dimensional characteristics x and y. Individuals live in continuous time, with a common discount rate r. They can only search when they are single; they meet with (flow) probability ρ. Divorce is as exogenous as can be: matches are dissolved randomly with probability ρ. Let $W(x)$ be the value[21] of an unmarried woman of characteristic x, and $M(y)$ be that an unmarried man of characteristic y. If these two individuals meet, they can obtain a flow marital surplus $s(x, y)$ until their match is dissolved. Suppose that they agree to divide it as $u(x, y) + v(x, y) = s(x, y)$. Since the match is dissolved with probability δ and its utility is discounted at rate r, the value $W(x|y)$ of the match for woman x is the value of $u(x, y)$ in perpetuity, minus the expected value lost if the match is dissolved:

$$rW(x|y) = u(x, y) - \delta(W(x|y) - W(x)).$$

The term $W(x|y) - W(x)$, and its analog $M(y|x) - M(y)$ for man y, represent their shares of the surplus relative to their outside option (waiting for a new partner). Like most of the search literature, Shimer and Smith

[20] See Smith (2011) for a more detailed review of the theory, and Section 3.4.2 for empirical applications.

[21] The discounted expected utility.

(2000) assume that these shares are equal:

$$W(x|y) - W(x) = M(y|x) - M(y). \tag{22}$$

Since $u+v \equiv s$, combining these equations shows that the common value in (22) is

$$\frac{s(x,y) - rW(x) - rM(y)}{2(r+\delta)}.$$

Now consider the value of an unmarried woman. Since with probability ρ she will meet a partner y drawn randomly from the distribution f of unmarried men, $W(x)$ is given by

$$rW(x) = \frac{\rho}{2(r+\delta)} \int (s(x,y) - rW(x) - rM(y)) f(y)dy.$$

Similarly,

$$rM(y) = \frac{\rho}{2(r+\delta)} \int (s(x,y) - rW(x) - rM(y)) g(x)dx$$

if g is the distribution of unmarried women.

Given distributions f and g, these two equations define the functions W and M. They can be interpreted as *acceptability conditions*: woman x and man y will agree to match if and only if the integrands of the right-hand sides are positive, that is if an only if the surplus $s(x,y)$ exceeds the sum of the reservation flow utilities $rW(x)$ and $rM(y)$. We denote $\alpha(x,y) = 1$ if this is the case, and $\alpha(x,y) = 0$ otherwise.

The densities f and g are equilibrium objects, however. Suppose that the pdf of the characteristics of all women (married or not) is n and that of all men is m. Then the pdf of the characteristics of married women is $n - f$. Since their matches dissolve with probability δ, in steady-state the number of new matches must exactly compensate. With fully random meetings, an unmarried woman x will match with probability $\rho \int \alpha(x,y)g(y)dy$. Therefore we have the *flow balance* equations

$$\delta(n(x) - f(x)) = \rho \int \alpha(x,y)g(y)dy$$
$$\delta(m(y) - g(y)) = \rho \int \alpha(x,y)f(x)dx.$$

Since α depends on W and M, we end up with four functional equations in f, g, W, and M. Shimer and Smith (2000) showed that if the marital surplus s is strictly supermodular (or submodular) and it is regular enough, a steady-state equilibrium exists. Contrary to the frictionless case however, it may not exhibit assortative matching; this requires additional log-modularity conditions on the derivatives of s that ensure that the sets of partners that are acceptable to a given individual are convex.

The literature on search and matching in the labor market suggests several variants of this basic model. Just as workers may engage in on-the-job search, married individuals may be looking for another partner. This brings up complex issues about commitment and search intensity. More generally, much less is known about the matching function in marriage markets than on the labor market. The static separable models we tend to work with exhibit constant returns to scale by construction; it may not be the best assumption in a dynamic context. If the matching process exhibits increasing returns, the theory of search models suggests that multiple equilibria would exist[22]. Finally, productivity shocks in employment relationships have an analog in match quality shocks in marriage: the surplus $s(x, y)$ may be hit by positive or negative shocks. Negative shocks (or positive shocks to alternative matches) require a mutually agreeable renegotiation of the existing agreement. If this is not possible, divorce may ensue.

1.4.3 Incorporating Same-sex Marriage

So far we have been applying a bipartite matching model to the marriage market: every couple must consist of one woman and one man. As same-sex marriage has become legal in more and more countries across the world, it is important to allow for couples in which both partners have the same gender. If individuals are immutably heterosexual or homosexual, then the heterosexual marriage market and the two same-sex marriage markets are separated. The heterosexual marriage market can be analyzed as in the previous sections. The two same-sex marriage markets are non-bipartite. Gale and Shapley (1962) called such markets "roommate markets"; they showed that a stable matching may fail to exist[23]. The intuition is that in such a market, there are many more potential blocking coalitions as any two individuals may match. Chiappori, Galichon, and Salanié (2019) showed that as long as the market is large and utility is perfectly transferable, a stable matching always exists. Their paper also demonstrates a simple way to reformulate any such market as a bipartite market; identification and estimation results can then be translated from the latter to the former.

If preferences for the gender of the partner are not absolute, the

[22] See for instance Diamond (1982, 1984).

[23] While their model excluded transfers, later literature showed that this result extends to the TU world.

marriage market cannot be subdivided ex-ante. Gender then becomes an additional observable characteristic of each individual, that codetermines the marital surplus. Each individual has an observable type (g, x), where g denotes the gender; and a couple generates a surplus $S((g, x), (g', x'))$. Unlike in the bipartite case, S must be symmetric—permuting (g, x) and (g', x') should not change its value. The feasibility constraints also must be redefined, as each individual can stay single, marry a same-sex partner, or marry an other-sex partner. After these redefinitions, the whole marriage market can be analyzed as a roommate market.

1.5 Dynamic aspects

Useful as the static model just described may be, dynamic considerations become paramount in analyzing investments, divorce, and remarriage.

1.5.1 Pre-marital investments

We have so far taken the individual characteristics on which people match as given, while they often are the outcome of previous investments; human capital is a prime example. Since matching patterns and the resulting surplus allocation between spouses depend on prior investments, they must also form part of the individual incentives to invest. Chiappori, Iyigun, and Weiss (2009b) modeled individuals who formulate expectations about what the market for marriage will be in the future; these expectations drive their investment decisions, and have to be self-fulfilling in the usual sense that they must be compatible with the patterns generated by the aggregation of the individual investment decisions.

An abundant literature has been devoted to such premarital investments. A pervasive question relates to the efficiency of premarital investments. From a theoretical perspective, investments and matching form a two-stage game: each individual first invests in their own human capital, then enters a matching game whose stable matching depends on all human capital investments. Call this game G. Can we expect these investments to be socially efficient, given the non-cooperative nature of the first stage? Surprisingly, the answer is yes, as shown by the early contributions of Peters and Siow (2002) in the case of large markets, and Iyigun and Walsh (2007a). A general argument has been provided by Cole, Mailath, and Postlewaite (2001) and Nöldeke and Samuelson (2015). The latter contribution can be summarized as follows. Consider an auxiliary game G_R in which the timing is reversed: individuals first match, then couples decide how to invest in human capital. Clearly, the outcome of G_R will be socially efficient. Nöldeke and Samuelson (2015) showed that the stable matching of G_R can always be implemented as a Nash equilibrium of G. As a consequence, the non-cooperative game G always has a Nash equilibrium that generates socially efficient investments[24].

It is important to note that the efficiency conclusion heavily relies on the equilibrium perspective that characterizes matching models of the marriage market. This contrasts with the approach adopted by the Iyigun and Walsh (2007b) or Basu (2006), who consider a household's bargaining process in isolation from the marriage market and conclude that endogenous reservation utilities typically generate inefficiencies.

1.5.2 The commitment issue

As discussed in Section 1.1, the imperfections or incompleteness of financial markets leave a crucial role to more informal interactions between members, aimed at facilitating transfers across periods or states of the world. Such mechanisms, however, require that individuals be able to commit to some future and/or contingent behavior. Risk sharing only works if luckier members can be trusted to compensate their less fortunate peers; similarly, lenders must be confident that borrowers will repay informal loans. Such informal agreements may be difficult to enforce, which in turn affects individuals' ability to reach agreements to start with. Generally speaking, any analysis involving intertemporal or contingent transfers must rely on specific assumptions on the level of commitment that can be expected to prevail between individuals. In turn, commitment issues must affect matching patterns, particularly when risk sharing or intertemporal transfers potentially constitute a major component of the marital surplus.[25]

Existing models can be classified into three main groups, depending on their treatment of commitment issues.[26] *Full Intrafamily Commitment* (FIC) models assume that individuals can, when matching, fully commit on their future behavior. While this enables them to conclude agreements that are efficient in an ex ante sense, full commitment is a very strong and often unrealistic requirement. Take the dissolution of couples, for instance. Full commitment does not necessarily preclude divorce or separation. As we shall see below, these decisions can be ex post efficient, and therefore be part of an ex ante efficient contract. But the ex ante agreement must then fully define the post-divorce out-

[24]Other equilibria may exist. For instance, if the surplus is supermodular and all agents but one underinvest, the last person's incentives to invest are suboptimal, which may lead to coordination failure.

[25]Lundberg and Pollak (2003) provide an early discussion of commitment issues and of their impact on marital patterns.

[26]The reader is referred to Chiappori and Mazzocco (2017) for a more detailed discussion.

comes. Moreover, the corresponding clauses will typically be contingent on the whole history of the relationship, including events that may not be observable by third parties. In addition, FIC excludes renegotiation. In particular, in any situation where the initial agreement does not entail separation, individuals are implicitly committed not only to remain married, but also to refrain from using the threat of divorce to achieve a better deal—irrespective of the context and in particular of the prevailing divorce legislation.

At the exact opposite, *No Intrafamily Commitment* (NIC) models assume that family decisions, including public goods and the allocation of private expenditures across members, are renegotiated at each period. As a result, agreements fail to be ex ante efficient; risk sharing opportunities are reduced and intertemporal transfers are severely limited. Decisions may even be inefficient in the ex post sense; for instance, agents may overinvest in human capital in order to improve their bargaining position in the future[27].

In the *Bargaining In Marriage* framework of Lundberg and Pollak (2003), no commitment is possible at all. Any promise made before marriage can be reneged upon just after the ceremony; there is simply no way spouses can commit beforehand to any future behavior. Moreover, upfront payments, whereby an individual transfers some money, commodities or property rights to the potential spouse conditional on marriage, are also excluded. This rules out mechanisms that restore dynamic efficiency by optimally setting up bargaining powers for the following period[28]. Then the intrahousehold allocation of welfare will be decided after marriage, irrespective of the commitment made before; and marriage decisions must anticipate the outcome of this future bargaining process. Importantly, this implies that matching models under NIC must adopt a non transferable utility setting in which each partner's share of the surplus is fixed and cannot be altered by transfers decided ex ante.

Mazzocco (2004) proposed an elegant, intermediate form of commitment. Building on previous work by Thomas and Worrall (1988) and Kocherlakota (1996) among others, the *Limited Intrafamily Commitment* (LIC) approach recognizes the existence of limits to individuals' ability to commit. An individual cannot legally commit not to divorce,

[27] In Basu (2006), the family maximizes a weighted sum of individual utilities. The weights depend on individual earnings, which are endogenous. As a result, individuals tend to supply too much labor. Basu analyzes the consequences of this situation, notably in terms of child labor.

[28] When such mechanisms can be used, the ability to commit for only *one* period ahead only may be sufficient to reach a full commitment optimum. See for instance Rey and Salanié (1990, 1996).

for instance. However, these limitations are now introduced as explicit constraints that reduce the set of feasible agreements. Agents will reach second-best efficient outcomes, subject to these constraints.

The simplest way to contrast these three models is (assuming away ex post inefficiencies) in terms of the dynamics of the Pareto weights. In a FIC context, the weights are determined at the date of marriage; they remain constant, including after divorce. The NIC framework has the opposite property: the weights are determined anew after even minor changes in the economic environment. The dynamics are more complex in the LIC model, where the Pareto weights follow a Markovian process. As long as no commitment constraint is violated, they remain constant, as required by ex ante efficiency. Should one constraint (say, person A's) be violated, then the Pareto weight of A is increased by the minimum amount needed for the constraint to become exactly binding. If this change results in the constraint of B being violated, then separation occurs. If not, the new Pareto weights are adopted and remain valid until the next time a commitment constraint binds[29].

1.5.3 Dynamic matching and divorce

Individuals are generally free to leave an existing relationship through divorce or separation, under conditions that vary with the legal and cultural environment. The existence of such outside options unavoidably affect the ability to commit. Any reform of the legislation governing divorce and separation potentially influences not only the number of divorces and the well-being of divorcees, but also individuals' behavior when married, and ultimately marital choices and premarital investments.

When do people divorce? While divorce has been abundantly studied, only recently has it been incorporated into the general framework of a mating market. Most models of divorce share a common structure. At each period, the household is affected by a random, non monetary shock that can be interpreted as a realization of the "quality of the match". If the shock is sufficiently negative, then each partner trades off the benefits of staying within the marital relationship with the potential gain resulting from breaking up, finding another partner, and drawing a new match quality shock. Both types of gains are computed as the expected present value of individuals' future trajectories, that is as the value function of a dynamic optimization program. Needless to say, the value of divorce for any individual depends on the legal framework. Both the allocation of decision rights—unilateral divorce vs mutual consent—and

[29] We will describe an empirical application by Lise and Yamada (2019) in Section 3.4.1.

the rules on alimony, custody, and control over the couple's assets play an important role in determining the post-divorce Pareto frontier.

The value of staying married, in turn, depends on the ability of the partners to renegotiate the terms of their relationship when facing a negative shock. If they can't, then divorce will sometimes be inefficient. It seems more reasonable to assume, as in the LIC framework, that spouses will not separate if some renegotiation of the current marital agreement could offer each of them a higher expected utility than they would get after divorce. Conversely, they will divorce if some renegotiation of the post-divorce allocation could result in both spouses being better off than remaining married under a renegotiated agreement. In both cases, the decision to divorce or not will be ex post efficient.

This argument can be summarized by the following figures, borrowed from Chiappori, Iyigun, and Weiss (2009a), where individual utilities are on the horizontal and vertical axes respectively. Point M (resp. D) denotes the current division of surplus if individuals remain married (resp. divorce). The red (resp. blue) line represents the Pareto frontier, i.e. the set of utility pairs that can be reached through transfers if spouses remain married (resp. divorce). Here:

- in Figure 1a, point D belongs to the interior of the Pareto set when married (in red), while point M is located outside of the Pareto set after divorce (in blue). As a result, divorce is inefficient, and partners remain married, perhaps after renegotiating the existing agreement.

- Figure 1b illustrates the opposite situation. Here point M belongs to the interior of the Pareto set when divorced, while D is outside the Pareto set if married. As a result, remaining married is inefficient, and individuals divorce; again, this may require a renegotiation of the post-divorce allocation.

Finally, recall that in the TU model all Pareto frontiers are straight lines with slope -1; therefore they cannot intersect. If partners can transfer utility to each other without cost and at a constant exchange rate *both* in marriage and after divorce, then one of the two Pareto sets must be included in the other. As a consequence, *all* Pareto efficient allocations belong to the higher Pareto frontier, and that will determine the divorce decision; the latter thus depends neither on divorce laws nor on asset allocation after divorce. This is the well-known Becker-Coase theorem. Reforms of divorce legislation (e.g., switching from mutual consent to unilateral, or changing post-divorce division of assets) may affect allocation between spouses both when married and after divorce, but not the divorce decision itself.

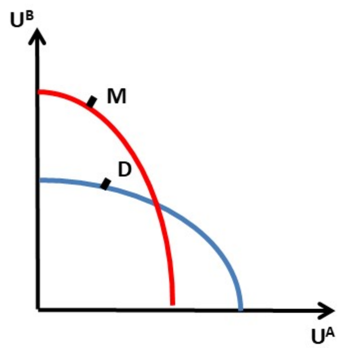

Figure 1a: spouses remain married

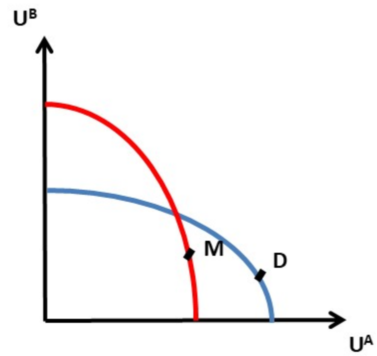

Figure 1b: spouses always divorce

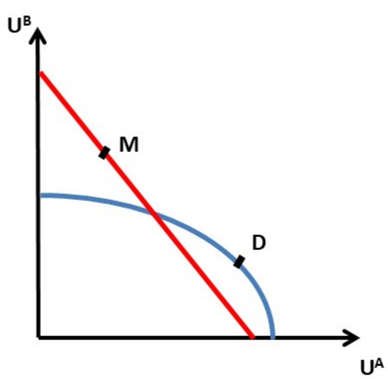

Figure 2a: divorce if unilateral only

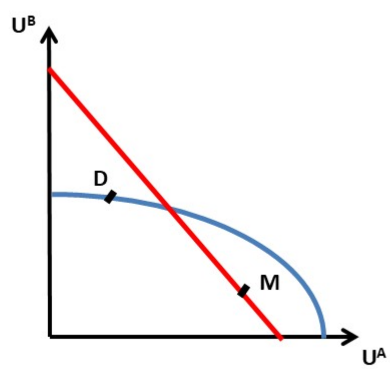

Figure 2b: divorce if mutual consent only

Source: Chiappori, Iyigun & Weiss 2009

This conclusion, however, requires efficiency *plus* the much stronger requirement of TU both before and after divorce. In particular, Figures 2a and 2b depict a situation in which TU prevails when married, but not after divorce—for instance because of public goods like children[30]. Then the Pareto frontiers may intersect; some efficient allocations require that spouses remain married, others that they divorce; and divorce laws have a direct impact on divorce decisions. For instance, in Figure 2a, spouses separate if the law allows unilateral divorce, but not if mutual consent is required; more surprisingly, Figure 2b displays the opposite pattern.

Post-divorce asset allocation The division of assets is an important element of divorce decisions. It clearly depends on the legislation governing divorce. New spouses may also be able to contract prenuptial agreements that cover, among other things, the allocation of household assets in case of divorce.

Existing laws can be taken as exogenous at the household level. They typically restrain individuals' ability to commit. In turn, these restrictions affect matching patterns. Take the following, two period example under TU. In period 1, women and men match according to some unidimensional characteristics x and y. In period 2, an exogenous match quality shock θ is realized[31]. Individuals may decide to remain married; then total utility is the sum of θ and some deterministic, economic component $s(x, y)$, which is assumed increasing in x and y and supermodular. In particular, if it were not for the possibility of divorce, matching would unambiguously be positive assortative. Alternatively, the partners may divorce; their respective utilities are then $U(x, y)$ and $V(x, y)$, where for simplicity we assume TU after divorce as well. Note that U and V depend not only on individual preferences, but also on the post-divorce allocation imposed by the legal system.

Agents divorce if and only if the shock is negative enough:

$$\theta + s(x, y) \leq U(x, y) + V(x, y)$$

or equivalently

$$\theta \leq \bar{\theta}(x, y), \text{ where } \bar{\theta}(x, y) = -s(x, y) + U(x, y) + V(x, y)$$

In the first period, matching decisions depend on the ex ante expected

[30]See Chiappori, Iyigun, and Weiss (2009a) for a detailed analysis.

[31]Some versions entail two shocks θ^A and θ^B (one for each spouse), thus allowing individuals' perceptions of the quality of the match to differ. Most of the following analysis remains valid by simply defining $\theta = \theta^A + \theta^B$.

surplus, which equals

$$ES(x,y) \equiv s(x,y) + F_\theta\left[\bar{\theta}(x,y)\right]\bar{\theta}(x,y) + \left(1 - F_\theta\left[\bar{\theta}(x,y)\right]\right) E\left(\theta \mid \theta \geq \bar{\theta}(x,y)\right)$$

where F_θ denotes the cdf of θ and f_θ its density. If ES is supermodular, individuals match assortatively in the first period. The sign of $\partial^2 ES/\partial x \partial y$ depends on the sign of the cross derivatives of U and V, as well as on the signs of the first derivatives of $\bar{\theta}$[32]. For given individual preferences, either positive or negative assortative matching may result, depending on the rules that govern the post-divorce division of assets. Clearly, any theoretical analysis of divorce decisions must rely on a structural model of household behavior.

Legal rules may prevent spouses from achieving ex ante efficiency. To see this, recall that ex ante efficiency is defined by Pareto weights remaining constant through time and across all states of the world. Now assume that the law stipulates that in case of divorce, all assets go to agent A. While this allocation is ex post efficient, its implicit Pareto weights, which strongly favor A, are unlikely to coincide with the weights prevailing during marriage. As a matter of fact, we would expect A's dominant position after divorce to be somewhat compensated by an allocation during marriage that favors B[33]. These features, in turn, will affect the initial matching game, possibly resulting in different matching patterns.

Prenuptial contracts, if available and legally enforced, ideally allow spouses to specify ex ante the Pareto weights that will prevail in all states of the world, including after divorce. As such, they may allow agents to achieve ex ante efficiency, blurring the previous distinction between FIC and LIC models[34]. The practical importance of prenuptial agreements

[32] More precisely,

$$\frac{\partial^2 ES}{\partial x \partial y} = \frac{\partial^2 s}{\partial x \partial y} + \frac{\partial^2 \bar{\theta}}{\partial x \partial y} + f_\theta \frac{\partial \bar{\theta}}{\partial x}\frac{\partial \bar{\theta}}{\partial y}$$

where f_θ denotes the cdf of θ. A sufficient condition for positive assortative matching thus is that $\bar{\theta}$ be supermodular and either increasing in x and y or decreasing in both variables.

[33] See for instance Chiappori, Iyigun, Lafortune, and Weiss (2017) for a detailed investigation.

[34] Ex ante efficient prenuptial contracts may, in some states of the world, entail contingent allocations that are ex post efficient but rely on different Pareto weights than those required by ex ante efficiency. These allocations, however, are exclusively realized out of the equilibrium path, where the deviation from ex ante efficiency is used to prevent individuals from reaching the corresponding nodes. The interested reader is referred to Chiappori, Costa-Dias, Meghir, and Xiao (2021) for a detailed analysis.

has been intensely debated in the literature. Prenuptial contracts do exist in many countries, and there is a reasonable expectation that their clauses will be enforced by the courts if and when divorce takes place. Still, most marriages do not use a prenuptial. The interpretation of this empirical fact is ambiguous: it may mean either that individuals are unable to write them (although it is not clear why), or that they do not need formal agreements of this type, presumably because the commitment devices already available are sufficient. An alternative explanation is that the optimal prenuptial contract should in principle be contingent on some variables (e.g. the match quality) that may not be observable by a third party.

Divorce and public goods As discussed in Section 1.1, public goods are a source of gains from marriage. In case of divorce, however, they raise specific issues that have often been overlooked by the literature. Some commodities are publicly consumed during marriage but not after divorce; for instance, spouses typically share housing while married but not once separated.[35] But other "goods" remain public even after divorce. Children expenditures are a typical example: most divorcees keep contributing to child costs.

At first glance, it would seem natural to posit that after a divorce, decisions on public goods are made in a non-cooperative way: while each parent keeps contributing to children expenditures, each of them simply stops taking into account the impact on the utility of the ex-spouse. In this view, cooperation would be a characteristic of the marital relationship, and would stop after its dissolution. This approach, however, is fraught with difficulties. Non-cooperative decisions regarding a public good generate a private contribution game. In a classic paper, Bergstrom, Blume, and Varian (1986) considered a model with a unique public and a unique private good. They showed that the private contribution game can admit only two types of equilibria. In the first type, a corner solution obtains: one individual totally stops contributing to the public good, which is entirely funded by the ex-partner. Such situations do exist—typically, the mother takes care of the children without any help from the father. They can hardly be considered as the norm however, let alone the only possible outcome.

The second type of solutions has an interior equilibrium, where both divorced parents contribute to the well-being of their children. However, the main result in Bergstrom, Blume, and Varian (1986) states that such equilibria exhibis a strong income pooling property: all public and pri-

[35] In this case, however, the TU property typically stops being satisfied after divorce, except for quasi linear preferences; see Chiappori, Iyigun, and Weiss (2009b).

vate consumptions of the ex-spouses only depend on their aggregate joint resources, not on each individual's income. For instance, paying a benefit to the husband instead of the wife would affect neither expenditures on children nor even the ex-spouses own consumption. In an interior, non cooperative equilibrium, any such transfer would be undone by a dollar-per-dollar increase in his expenditures on children—and a corresponding reduction of her contribution. In other words, adopting a non cooperative framework would amount to assuming that ex-spouses start pooling their individual incomes after divorce. This prediction sounds strikingly counterfactual.

Bergstrom, Blume, and Varian (1986) extended their income pooling result to any number of public goods. Furthermore, Browning, Chiappori, and Lechene (2010) show that there can be at most one public good to which ex spouses both contribute. All other public goods must be exclusively taken in charge either by the ex-husband or by the ex-wife[36]. Thus, should one consider the child's health, education, entertainment, sport activities, etc. as different public goods, if both parents contribute to (say) health then it cannot be the case that they both contribute to any other outcome. Similarly, if there are two children or more, if parents both contribute to the first child then it cannot be the case that they both contribute to any other child. Finally, if there is indeed one public good to which they both contribute, then the same income pooling result as before obtains: ex-spouses individual consumptions only depend on the sum of their individual incomes.

In a nutshell, assuming non cooperation after divorce does not sound like a very promising avenue. In an early contribution, Weiss and Willis (1985) assumed that households agree on basic elements of a divorce settlement when they marry. However, this framework also generates corner solutions, with the non-custodial ex-spouse only contributing what is required by the settlement. Alternatively, Chiappori, Costa-Dias, Meghir, and Xiao (2021) maintain the efficiency assumption after divorce, but with different Pareto weights which may or may not have been set ex ante (e.g. as part of a prenuptial agreement). This returns us to our previous discussions of commitment and the dynamics of Pareto weights after divorce.

1.5.4 Remarriage

So far we have represented post-divorce individual utilities in a reduced form manner (the functions U and V of page 35). If individuals remain

[36]This conclusion has been extended by Doepke and Tertilt (2019) to domestic production. Their model has a continuum of public goods, none of which is jointly produced in equilibrium.

single forever after divorce, these functions can be seen as the present expected value of an individual's future utility. Remarriage introduces new and difficult issues. The expected utility of an ex-spouse, which plays a fundamental role in the decision to divorce, depends on the probability of remarriage but also on the division of the surplus between spouses that will prevail in the new household. The latter, however, is driven by the distributions of potential new spouses, which in itself depends on individual divorce decisions. In other words, divorce becomes an *equilibrium* phenomenon, in which individual divorce decisions, based on expectations regarding the state of the market for (re)marriage, generate a matching game, the outcomes of which have to fulfill these expectations[37]. The fact that so many divorcees who remarry[38] do so with a partner who never married before only complicates matters further.

One possible approach is to construct a dynamic model in which individuals can move between two states (single and married), and to characterize the stationary equilibria of such models. In particular, divorcees simply move back to the stock of unmarried people, and are therefore part of the dynamic search process from that moment on. This path has recently been followed in models based on a search framework[39]. A number of interesting questions remain; for instance, whether a divorcee and a never married individual with identical characteristics should be considered as perfect substitutes.

1.5.5 Marriage versus cohabitation

A striking feature of mating patterns over recent decades is, in many countries, a sharp decline in the number of marriages, often accompanied by a significant increase in cohabitation. The distinction between marriage and cohabitation is complex; many of the economic gains generated by marriage (starting with sharing public goods such as housing) are also generated by cohabitation. Moreover, as argued by Ciscato and Weber (2020), cohabitation out of wedlock can be a "trial period" before marriage but also an alternative to it; distinguishing between these two situations is empirically challenging, especially since the frontier between them fluctuates with time and across economies.

A large literature has been devoted to analyzing trends in marriage versus cohabitation (see for instance Bumpass and Sweet (1989), Choo and Siow (2006), Mourifié and Siow (2017), Qian and Preston 1993 and Schwartz (2010), to name just a few). Moreover, several contributions

[37] This is similar to marriage with premarital investment, as described in Section 1.5.1.

[38] About half in the US.

[39] See for instance Goussé, Jacquemet, and Robin (2017), to which we will return in Section 3.4.2.

have addressed the structural underpinnings of these trends. Most of the existing literature recognizes that marriage involves a higher degree of commitment than cohabitation - a property that may be crucial when it comes to incentives to invest into the relationship. Lundberg, Pollak, and Stearns (2016) thus argue that marriage is used by college-educated couples as a commitment device that enables more efficient investments on children, while cohabitation (and the resulting family instability) becomes increasingly frequent for less educated individuals. In a similar vein, Lafortune and Low (2017) recognize that commitment issues can seriously hamper spouses' incentives to invest; the efficiency of marriage as a commitment device stems from the fact that, unlike cohabitation, separation from marriage typically generates an equal division of assets, particularly of housing. However, this mechanism is only available to people with sufficient assets; the latter are thus more likely to marry, while poorer folks tend to have more early extramarital fertility.

2 Empirical Methods

In marriage markets, the data the econometrician observes typically consists of a list of matches, along with some characteristics of the partners in each match: "who matches whom", as per our question **Q1**. It might record, for instance, the number of marriages of college-educated men born in 1967 with female high-school graduates born in 1968. The data would often also tell us how many such men and women remained single. Sometimes more information is available: the number of children, divorces, remarriages. In principle, they might be used as proxies for the joint surplus of a match. They have rarely been used for that purpose in mating markets, however, as they are likely to be very noisy proxies for the expected joint surplus when the partners decided to marry.

Men and women obviously do not only match on characteristics like age and education, which are typically observed by the econometrician. Mutual attraction depends on a host of traits that are observable to the potential partners but are not recorded in the data. Even in one-sided markets, the appeal of a given product for a given consumer depends on unobservable variation in tastes. In marriage markets, this difficulty is compounded by the existence of unobserved variation in preferences on both sides, as well as in the many interactions that create marital surplus.

The importance of this two-sided unobserved heterogeneity is both an opportunity and a challenge. It is an opportunity in that it allows us to reconcile the (too) stark predictions of theoretical models—such as positive assortative matching—with the wide variation in matching patterns that we observe among observably identical individuals. It is a challenge in the identification problem it generates. The literature has now largely converged on a "separable" approach that restricts the interaction between unobserved characteristics. Our discussion of empirical methods will focus on it.

While we will mostly describe frictionless matching models, we should note at this stage that the search framework provides a complementary explanation for the variability of matching patterns in the data. Since meetings are random and infrequent, identical individuals will face different sequences of potential partners and end up in different matches, even in the absence of unobserved variation in the marital surplus. Our position is that search models are especially useful when describing the transitions between different marital states. We will discuss them in Section 3.4.2; until then, we take our data to be cross-sectional.

Up to now a woman had characteristics x and a man had characteristics y. We need to distinguish those characteristics that are observed by the econometrician and those that are not, and therefore constitute unobserved heterogeneity. With a mild abuse of notation, we will now let x and y denote the observed characteristics only, and we will call them the *type* of the individual. The *full type* $\tilde{x} = (x, \varepsilon)$ of a woman will also include unobserved heterogeneity ε. Similarly, the full type of a man $\tilde{y} = (y, \eta)$ includes his type y and his unobserved heterogeneity η. The hypothetical match of a woman of full type $\tilde{x}$ with a man of full type $\tilde{y}$ generates a marital surplus which we denote $\tilde{S}(\tilde{x}, \tilde{y})$.

To simplify the exposition, we will assume until Section 2.4.4 that the types x and y only take a finite number of values. On the other hand, the unobserved heterogeneity terms ε and η may be discrete or continuous, and they may have several dimensions. As before, we allow for singlehood and we denote $X := \mathcal{X} \cup \{\emptyset_X\}$ and $Y := \mathcal{Y} \cup \{\emptyset_Y\}$. Finally, we let $(\mu_{xy})_{(x,y) \in X \times Y}$ denote the numbers of matches in a "cell" of types (x, y).

2.1 The Separable Approach

The marital surplus $\tilde{S}(\tilde{x}, \tilde{y})$ a priori may interact four groups of arguments: the observed characteristics x, y and the unobservable heterogeneities ε and η. Separability rules out any interaction between ε and η:

Assumption 10 (Separability) *The joint utility of a match between $\tilde{x} = (x, \varepsilon)$ and $\tilde{y} = (y, \eta)$ is*

$$\tilde{S}(\tilde{x}, \tilde{y}) = S_{xy} + \zeta_y(\tilde{x}) + \xi_x(\tilde{y}).$$

A single woman $\tilde{x}$ has utility

$$\tilde{S}(\tilde{x}, \emptyset_Y) = \zeta_0(\tilde{x})$$

and a single man $\tilde{y}$ has utility

$$\tilde{S}(\emptyset_X, \tilde{x}) = \xi_0\left(\tilde{y}\right).$$

Note that separability is a property of the *marital surplus* of a match, not of the pre-transfer utilities of the partners. Separable preferences clearly imply a separable joint surplus, but the converse is not true.

Separability has proved to be a very useful assumption. It was introduced by Choo and Siow (2006) and named by Chiappori, Salanié, and Weiss (2017), who derived its general implications. It has its pros and cons, naturally. To illustrate, suppose that the marital surplus of a match is higher when the partners worship the same religion. If religion is not recorded in the data, then by definition it goes into ε and η, and separability fails. If richer data becomes available and religion is observed, this would not be a concern any more.

Even if (in this example) religion is not observed, one might hope that assuming separability does not bias the most crucial estimates too much if religion is conditionally independent of the observed characteristics. Chiappori, Nguyen, and Salanié (2019) simulated such a non-separable model and took a separable model to the data that it generated. Their findings suggest that the resulting estimation bias on the (x, y) complementarities is surprisingly small.

In the end, imposing separability is a pragmatic choice, as it would require a lot of data and/or assumptions to reliably estimate the interactions between unobserved characteristics. It is important to emphasize that separability does not rule out "matching on unobservables". The following result, due to Chiappori, Salanié, and Weiss (2017), describes its implications:

Theorem 11 (Splitting the Surplus under Separability) *Under Assumption 10, there exists a pair of matrices (U, V) such that at any stable matching (μ_{xy}):*

- *a woman of full type $\tilde{x} = (x, \varepsilon)$ will match with a man of an observable type y that maximizes $U_{xy} + \zeta_y(x, \varepsilon)$ over X*
- *a man of full type $\tilde{y} = (y, \eta)$ will match with a woman of an observable type x that maximizes $V_{xy} + \xi_x(y, \eta)$ over Y*
- $U_{x0} = V_{0y} = 0$
- $U_{xy} + V_{xy} \geq S_{xy}$, *with equality if $\mu_{xy} > 0$.*

Proof. We know from Section 1.3.1 that the utility of woman $\tilde{x}$ at a stable matching is

$$\tilde{u}(\tilde{x}) = \max_{\tilde{y}}(\tilde{S}(\tilde{x}, \tilde{y}) - \tilde{v}(\tilde{y})).$$

Breaking down the maximization over y-then-η and using separability gives

$$\begin{aligned}\tilde{u}(\tilde{x}) &= \max_{y}\left(S_{xy} + \zeta_y(\tilde{x}) + \max_{\eta}\left(\xi_x(y,\eta) - \tilde{v}(y,\eta)\right)\right) \\ &= \max_{y}\left(S_{xy} + \zeta_y(\tilde{x}) - \min_{\eta}\left(\tilde{v}(y,\eta) - \xi_x(y,\eta)\right)\right).\end{aligned}$$

Denote $V_{xy} = \min_\eta\left(\tilde{v}(y,\eta) - \xi_x(y,\eta)\right)$; then

$$\tilde{u}(\tilde{x}) = \max_{y}\left(S_{xy} - V_{xy} + \zeta_y(\tilde{x})\right).$$

Similarly, we can define $U_{xy} = \min_\varepsilon\left(\tilde{u}(x,\varepsilon) - \zeta_y((x,\varepsilon)\right)$. The stability constraints $\tilde{u}(\tilde{x}) + \tilde{v}(\tilde{y}) \geq \tilde{S}(\tilde{x},\tilde{y})$ imply that $U_{xy} + V_{xy} \geq 0$. If $\mu_{xy} > 0$, then there exist $(\tilde{x},\tilde{y})$ such that $\tilde{u}(\tilde{x}) + \tilde{v}(\tilde{y}) = \tilde{S}(\tilde{x},\tilde{y})$; then $U_{xy} + V_{xy} = S_{xy}$. ■

The intuition of this result is simple. The term $\zeta_y(\tilde{x})$ can be seen as a contribution that $\tilde{x}$ brings to all matches she could establish with men of observable characteristics y. Just as a worker who is \$1 more productive than another in every job will get a \$1 higher wage in equilibrium, a woman with a higher value of ζ will reap its value in a stable matching.

Assuming separability greatly reduces the complexity of the matching problem: our unknown now is the matrix U, which is defined on the set of observable types rather than on the set of full types. With discrete x and y, the problem becomes finite-dimensional. Suppose that $\mu_{xy} > 0$ for all (x,y). Then given U, we can define $V = S - U$, and obtain the equilibrium utilities:

$$\tilde{u}(\tilde{x}) = \max_{y \in Y}(U_{xy} + \zeta_y(x,\varepsilon)) \tag{23}$$

and

$$\tilde{v}(\tilde{y}) = \max_{x \in X}(V_{xy} + \xi_x(y,\eta)). \tag{24}$$

Moreover, the maxima in these simple, one-sided discrete choice problems are achieved by the stable matching partners[40].

[40] When the maximum in (23) for instance is achieved at $\emptyset_Y$, woman $\tilde{x}$ remains single.

The vector $(U_{xy})_{y \in Y}$ represents a general propensity of women of type x to match with men of the different types; as (23) shows, this is combined with a specific propensity $(\zeta_y(x, \varepsilon))_{y \in Y}$ of women of full type (x, ε) to match with different types of men. At a stable matching, each woman is indifferent between all men of her preferred type y. This is a consequence of the separability assumption. She will not marry any man of type y, however. The man she ends up marrying will be one whose η gives a relatively high value to $\xi_x(y, \eta)$. In this sense, the matrix U drives matching over observables and the ζ and ξ terms drive matching over unobservables.

2.2 Identification of Separable Models

We assume that the data has information on the matching patterns μ_{xy}, including the number of singles[41]. This data allows the analyst to reconstruct the number of men and women with any observed characteristics: $n_x = \sum_{y \in Y} \mu_{xy}$ and $m_y = \sum_{x \in X} \mu_{xy}$.

The distributions of the ζ and ξ terms are not known, however. In their pioneering contribution, Choo and Siow (2006) assumed that these terms were drawn from independent and identically distributed standard type I extreme value distributions. As it turns out, the analysis of identification can be carried out for much more general distributions, and allow for rich correlations and heteroskedasticity. Consider the vector of random variables $(\zeta_y(x, \cdot))$ for $y \in Y$. We will denote its distribution as $\mathbb{P}_x$; and we denote $\mathbb{Q}_y$ the distribution of the random vector $(\xi_x(y, \cdot))$ for $x \in X$. In the Choo and Siow (2006) example, each $\mathbb{P}_x$ is a random vector of $|Y|$ iid standard type I EV variables.

As explained in Section 1.3.3, the stable matching solves an optimal transportation problem whose objective function is the total joint utility generated by a matching:

$$\mathcal{W} = \int \tilde{u}(\tilde{x}) \tilde{n}(d\tilde{x}) + \int \tilde{v}(\tilde{y}) \tilde{m}(d\tilde{y}). \tag{25}$$

We will simply call it the *social welfare* from now on. The dual formulation of the matching problem states that $\mathcal{W}$ must be minimized under the stability constraints

$$\tilde{u}(\tilde{x}) + \tilde{v}(\tilde{y}) \geq \tilde{S}(\tilde{x}, \tilde{y}).$$

Galichon and Salanié (2021a) showed that in any separable model, the

[41]What follows could be adapted if the data only pertains to couples. The main difference is that the matrix (S_{xy}) could only be identified up to arbitrary additive components $a_x + b_y$.

social welfare can be rewritten as follows:

$$\mathcal{W}(S) = \max_{\mu} \left(\sum_{x,y} \mu_{xy} S_{xy} + \mathcal{E}(\mu) \right) \tag{26}$$

where the *generalized entropy* $\mathcal{E}$ is a function whose shape only depends on the distributions $\mathbb{P}_x$ and $\mathcal{Q}_y$.

The maximand in (26) consists of two terms. The first one is the value of social welfare if partners only matched on the basis of their observable types. Unobserved heterogeneity generates matching on unobservables, which adds another contribution to the social welfare $\mathcal{W}$ via the generalized entropy term.

Taking the first-order conditions in this problem gives

$$S_{xy} = -\frac{\partial \mathcal{E}}{\partial \mu_{xy}}(\mu). \tag{27}$$

Since the matching patterns μ are recorded in the data, for any choice of the distributions $\mathbb{P}_x$ and $\mathcal{Q}_y$ this equation identifies the matrix (S_{xy}). We obtain nonparametric identification of S *conditional* on knowing (or assuming) the distribution of the unobserved heterogeneity. To put it differently: for any assumed distribution of the ζ and ξ terms, for any observed matching patterns μ, there exists a matrix S that rationalizes μ.

While this may seem disappointing, it is not that surprising: we only observe $(|X| \times |Y| - 1)$ numbers (the μ_{xy}). Unless we restrict the parameterization unknown matrix S, we just have too little information to learn about the distributions of unobserved heterogeneity, or to test the model. If we do use a low-dimension parameter vector for the matrix S, then we may use other degrees of freedom to parameterize the ζ and ξ, and generate testable predictions. Another (and complementary) option is to pool data from several marriage markets and to assume that some elements of the specification are constant across markets.

2.3 The Logit Model

The most popular specification of the separable model is the multinomial logit of Choo and Siow (2006), which Choo (2015) extended in a multiperiod framework. As already mentioned, Choo and Siow (2006) assumed that x and y take discrete values and that

$$\zeta_y(x, \varepsilon) = \varepsilon_y \quad \text{and} \quad \xi_x(y, \eta) = \eta_x$$

where the vectors (ε_y) and (η_x) are drawn from standardized type-I extreme value distributions.

In this case, the generalized entropy $\mathcal{E}$ is simply the standard entropy

$$\mathcal{E}(\mu; n, m) = -\sum_{x \neq 0, y \neq 0} \mu_{xy} \log \frac{\mu_{xy}^2}{n_x m_y} - \sum_{x \neq 0} \mu_{x0} \log \frac{\mu_{x0}}{n_x} - \sum_{y \neq 0} \mu_{0y} \log \frac{\mu_{0y}}{m_y}.$$

and equation (27) gives the very simple *Choo and Siow formula*

$$S_{xy} = \log \frac{\mu_{xy}^2}{\mu_{x0}\mu_{0y}}. \tag{28}$$

Since it has no free distributional parameter, the logit specification circumvents the identification issues mentioned in Section 2.2. On the other hand, it suffers from the usual issues of the multinomial logit: it has very constrained comparative statics, and relabeling the types has spurious effects[42]. These problems all stem from the assumption that the unobservable shocks are independent across potential partners[43]. Richer specifications would allow for "local" correlation structures.

2.4 Estimation of Separable Models

The data typically consists of a large sample of N households. Of those, $\hat{\mu}_{xy}$ are marriages between types x and y; $\hat{\mu}_{x0}$ are single women of type x, and $\hat{\mu}_{0y}$ are single men of type y. These natural estimates of the matching patterns μ generate margins

$$\hat{n}_x = \sum_y \hat{\mu}_{xy} + \hat{\mu}_{x0}$$
$$\hat{m}_y = \sum_x \hat{\mu}_{xy} + \hat{\mu}_{0y}.$$

The estimators $\hat{\mu}$ are distributed as discrete count variables. If the N households are drawn with equal probabilities from an infinite population characterized by true matching patterns μ, then

$$\text{cov}(\hat{\mu}_{xy}, \hat{\mu}_{zt}) = \frac{1}{N}\mu_{xy}(\mathbb{1}(x = z, y = t) - \mu_{zt}).$$

The data often come with sampling weights, which are easily accommodated.

2.4.1 Nonparametric Estimation of the Surplus

If the distributions $\mathbb{P}_x$ and $\mathbb{Q}_y$ are parameter-free and the generalized entropy function $\mathcal{E}$ is easy to evaluate, then one can use (27) directly

[42]See Galichon and Salanié (2021a) for a longer discussion.

[43]For instance, ε_y and ε_t are independent if $y \neq t$.

to obtain a nonparametric estimator $\hat{S}$ of the surplus matrix. The estimator $\hat{S}$ is $\sqrt{N}$-consistent and asymptotically normal; its asymptotic distribution of follows directly from that of the estimated matching patterns $\hat{\mu}$. The logit model is a leading example; Choo and Siow (2006) used (28) to estimate the surplus.

If the error distributions are not fully-specified, then the model is underidentified unless restrictions are imposed on the specification of the surplus matrix S and/or more data is used (see Section 3.3 for the latter).

2.4.2 Parametric Estimation

We assume here that the data was generated by a fully parametric model for some unknown parameter vector θ_0. Some of the components of θ_0 may be used to parameterize the matrix S and others may for instance be shape parameters for the distributions $\mathbb{P}_x$ and $\mathbb{Q}_y$. Some of what follows is specialized to an important special case, in which the surplus function S is linear in the unknown parameters:

$$S_{xy}^{\theta} = \sum_{k=1}^{K} \theta_k s_{xy}^k \tag{29}$$

where the s^k are known basis functions. We call this the *semilinear* model.

Maximum Likelihood Estimation The most generally applicable way to estimate a parametric separable matching model is maximum likelihood. Suppose that we know how to compute the stable matching μ^θ for any given value of θ—we could use (25), but there are often much faster alternatives[44].

Note that this results in a number of households that typically differs from the observed N:

$$N^\theta = \sum_{x,y} \mu_{xy}^\theta + \sum_x \mu_{x0}^\theta + \sum_y \mu_{0y}^\theta.$$

The likelihood function of the sample is

$$\log L(\theta) = \sum_x \sum_y \hat{\mu}_{xy} \log \frac{\mu_{xy}^\theta}{N^\theta} + \sum_x \hat{\mu}_{x0} \log \frac{\mu_{x0}^\theta}{N^\theta} + \sum_y \hat{\mu}_{0y} \log \frac{\mu_{0y}^\theta}{N^\theta}.$$

The estimator given by the maximization of $\log L$ has the usual properties: it is $\sqrt{N}$-consistent, asymptotically normal, and asymptotically efficient.

[44]See Galichon and Salanié (2021a) for a much more detailed discussion.

Minimum Distance Estimation Galichon and Salanié (2021b) show that any separable model can be estimated by minimizing the norm of the difference between the two sides of (26). While this may seem complicated, it boils down to a two-step quasi-generalized least-squares procedure if the model is semilinear. Moreover, it becomes one-step weighted least squares if the distribution of the error terms is parameter-free.

Moment Matching in Semilinear Models In a semilinear model, it seems tempting to match the observed *comoments* $\hat{C}^k = \sum_{x,y} \hat{\mu}_{xy} s^k_{xy}$ with their simulated counterparts:

$$\hat{C}^k = \sum_{x,y} \hat{\mu}_{xy} s^k_{xy} = \sum_{x,y} \mu^{\theta}_{xy} s^k_{xy}.$$

This results in K equations, which determine the K coefficients of the basis functions for fixed values of the parameters of the distributions $\mathbb{P}_x$ and $\mathbb{Q}_y$. Galichon and Salanié (2021a) show that the resulting estimators can be obtained by maximizing a globally concave function. If there are any distributional parameters, they can be optimized over in an outer loop. While the moment matching estimator is less efficient than the maximum likelihood estimator, it has a more direct intuition.

Estimating the Logit Model In the logit model of Section 2.3, one can avoid having to compute the stable matching (or evaluating the social welfare $\mathcal{W}$). Galichon and Salanié (2021b) show that in a semilinear logit model, the moment matching estimator can be obtained by a simple Poisson regression where the matching patterns play the role of the counts. The regression also yields estimates of the equilibrium utilities of all type as a by-product. It can be implemented easily using off-the-shelf software.

2.4.3 Index Models

Many covariates may a priori codetermine the joint surplus. Since these models are not easy to identify, it would be useful to be able to test whether a simple combination of covariates (an *index*) suffices to summarize the contribution of each partner to the joint surplus, as in Section 1.3.4. More precisely, can the surplus be written as

$$S(x, y) = S(I(x), J(y)) \tag{30}$$

for two scalar indices $I(x)$ and $J(y)$? The study of this class of models was initiated by Chiappori, Oreffice, and Quintana-Domeque (2012). They proved that in equilibrium, the conditional distribution of the index $I(x)$ given y, depends only on the index $J(y)$, and conversely. However, this property may not extend to the distribution of x conditional on y, making testing difficult in general (Chiappori, Oreffice, and Quintana-Domeque, 2020).

Fortunately, one can prove[45] that in the logit model, if the joint surplus satisfies (30) the distribution of x conditional on y only depends on the value of $J(y)$. One can therefore test the index property by regressing the various components of x on y and testing that the right-hand sides are proportional. This is especially straightforward if the index $J(y)$ is assumed to be linear, of course.

2.4.4 Continuous Observed Characteristics

We assumed so far that the types x and y were discrete. This is of course restrictive; several useful observed characteristics are continuous. Now continuous-choice models are not simply a limit of discrete-choice models. The expected utility of choosing between J similar alternatives, for instance, grows to infinity with J (in $\log(J)$ for the logit model.) One way around this issue is to require (quite reasonably) that couples meet before they can form a match, and to restrict the process that generates meetings. Dagsvik (1994) showed that if meetings are generated as the points of a Poisson process with a well-chosen intensity, this results in formulæ that are the continuous analogs of those Choo and Siow (2006) obtained for the discrete logit model of Section 2.3. Sums only need to replaced by integrals, and probability masses should be replaced with probability densities[46].

Dupuy and Galichon (2014) showed how the techniques described in previous subsections extend naturally to this continuous logit model. In addition, they developed very simple inference procedures for a quadratic specialization of this continuous logit model. Suppose that $S^\theta(x,y) = x'\theta y = \sum_{i=1}^n \sum_{j=1}^m \theta_{ij} x_i y_j$, where θ is an unknown "affinity matrix"[47]. This model nests many specifications that can be selected by estimating the rank of the matrix θ. If for instance θ is a rank one matrix, then it can be written as $\theta = ab'$, where a and b are two column vectors defined up to a multiplicative scalar. In that case, $g(x) = x'a$ and $h(y) = b'y$ can be interpreted as one-dimensional attractiveness indices, along the lines of Section 1.3.4. More generally, if θ is of rank r then $S^\theta(x,y)$ can be written as a sum of products of indices: $\sum_{k=1}^p (x'a_k)(b'_k y)$. Dupuy and Galichon (2014) show how θ can be estimated, and how to test for its rank.

[45]See Appendix A of Guadalupe, Rappoport, Salanié, and Thomas (2021).

[46]One can also combine discrete and continuous types.

[47]Bojilov and Galichon (2016) derive closed-form formulæ for the special case when the types x and y are normally distributed.

2.5 Maximum-score methods

In a series of papers starting with Fox (2010), Fox has developed an empirical approach to matching with transferable utility that relies on a selecting set of "matching inequalities." The intuition behind it is simple. Suppose that $\tilde{x}$ marries $\tilde{y}$ and $\tilde{x}'$ marries $\tilde{y'}$. If these two couples are part of a stable matching, then reshuffling partners cannot increase the sum of their surpluses:

$$\tilde{S}(\tilde{x}, \tilde{y}) + \tilde{S}(\tilde{x}', \tilde{y}') \geq \tilde{S}(\tilde{x}, \tilde{y}') + \tilde{S}(\tilde{x}', \tilde{y}).$$

If we observe C couples $(\tilde{x}_i, \tilde{y}_i)$ and we assume that it belonged to a stable matching generated by a surplus $\tilde{S}^\theta(\tilde{x}_i, \tilde{y}_j) \equiv \tilde{S}^\theta_{ij}$, we could write

$$\sum_{i<j} (\tilde{S}^\theta_{ii} + \tilde{S}^\theta_{jj} - \tilde{S}^\theta_{ij} - \tilde{S}^\theta_{ji}) \geq 0.$$

Under reasonable conditions, only a small set of values of θ would satisfy all of these inequalities.

This is of course not a feasible approach in practice: we never observe matching between full types $\tilde{x}$ and $\tilde{y}$, only between types x and y. Now it is easy to see that in the logit model of Section 2.3, (28) implies that if we observe the couples (x, y) and (x', y'),

$$S_{xy} + S_{x'y'} - S_{xy'} - S_{x'y} = 2 \left(\log \mu_{xy} + \log \mu_{x'y'} - \log \mu_{xy'} - \log \mu_{x'y} \right). \tag{31}$$

Graham (2011, 2014) proved that if the unobserved heterogeneity terms ζ and ξ are independently and identically distributed, then the two sides of (31) must have the same sign.

Fox (2010) called this the *rank-order property* and Fox (2018) weakened the iid requirement to exchangeability. While this set of inequalities is less informative than (and is implied by) (31), it is valid in a much larger set of models. On the other hand, it does not allow for nested logit or mixed logit structures.

Now consider the function

$$F(\theta) \equiv \sum_{i<j} \mathbb{1} \left(S^\theta_{ii} + S^\theta_{jj} - S^\theta_{ij} - S^\theta_{ji} > 0 \right)$$

where i and j range over the set of observed matches. Much like in Manski (1975), maximizing $F(\theta)$ gives a set-valued estimator of θ that converges to a set that includes the true parameter value.

Note that instead of summing over all ordered pairs $i < j$, we could select a subset of inequalities that seem particularly relevant or informative. This may allow for more robust inference. On the other hand, the maximum-score method minimizes a discontinuous function and only yields a set-valued estimator; and it only applies to models with exchangeable error distributions.

3 Some empirical applications

Many empirical applications of mating models have appeared over the recent years; we only present here a small selection.

3.1 Measuring homogamy

In the analysis of matching patterns, the notion of *homogamy* is of particular interest: to what extent do individuals marry their own kind, and what are the economic consequences? Assortative matching mechanically increases inequality across households, relative to random matching. Its long term economic implications are even more critical. Educated parents tend to invest more (and more efficiently) in their children's education; the final outcome could be an "inequality spiral" (Chiappori, 2017), whereby at each generation, children born from parents with high human capital get further ahead of other children. Any increase in preferences for homogamy may therefore have a large impact on the *dynamics* of inequality[48].

Analyzing the evolution of homogamy, however, raises challenging issues when the marginal distributions of match-relevant characteristics change over time. As many more women graduate from college, one would naturally expect an increase in the number of couples where both spouses are college graduates. This mechanical impact would happen even if the surplus from any match remained constant, which is highly unlikely. Theory predicts for instance that individuals should be more willing to match assortatively when human capital becomes more valuable, which boosts the returns to parental investment on children[49].

To illustrate how these different factors complicate the measure of homogamy, take the following, simple example in which an equal mass (normalized to 1) of men and women are distributed into two classes: Educated and Uneducated. Assume that the surplus from any match is large enough that no one remains single in equilibrium. Matching patterns in this population then are fully described by a 2×2 table:

In Table 1, m and n are the proportions of Educated females and

[48] Lundberg, Pollak, and Stearns (2016) argue that marriage is used by college-educated couples as a commitment device that enables more efficient investments on children, while cohabitation (and the resulting family instability) becomes increasingly frequent for less educated individuals. This tendency reinforces the inequality trend.

[49] See Chiappori, Salanié, and Weiss (2017).

Table 1: Matching by education

${}_{w}\backslash^{m}$	Educated	Uneducated
Educated	$p_{EE} = r$	$p_{EU} = m - r$
Uneducated	$p_{UE} = n - r$	$p_{UU} = 1 - m - n + r$

males, and $p_{EE} = r$ is the proportion of couples where both spouses are Educated. It is easy to define assortative matching here. A (m, n, r) table of this type exhibits Positive Assortative Matching (PAM) if the proportion of couples with equal education (the sum of the diagonal cells of the table) is larger than what would obtain under random matching—that is if and only if

$$r \geq mn.$$

Now suppose we want to compare two tables (m, n, r) and (m', n', r'). If the marginals are the same ($m = m'$ and $n = n'$), the answer is clear: table (m, n, r) exhibits more preference for assortativeness than table (m, n, r') if and only if $r \geq r'$. The choice of an homogamy ranking is more difficult when the margins differ.

3.1.1 Existing indices

Many different criteria can be found in the literature; they may lead to different comparisons and rankings. The most widely used criterion relies on the "odds ratio" index:

$$I_O(m, n, r) = \ln \frac{p_{EE} p_{UU}}{p_{EU} p_{UE}} = \ln \left(\frac{r(1 - m - n + r)}{(m - r)(n - r)} \right).$$

This index is popular in the demographic literature, as it can be directly derived from a log-linear approach[50]. In economics, it has been used by Siow (2015), Chiappori, Salanié, and Weiss (2017), Ciscato and Weber (2020) and Chiappori, Costa-Dias, Crossman, and Meghir (2020) among many others.

Alternatively, several authors[51] use a linear regression framework. This leads them to use the correlation between wife's and husband's education:

$$I_{\text{Corr}}(m, n, r) = \frac{r - mn}{\sqrt{mn(1 - m)(1 - n)}}$$

[50] See for instance Mare (2001) and Schwartz and Mare (2005).

[51] Among which Greenwood, Guner, and Knowles (2003) and Greenwood et al. (2014).

In this 2×2 case, the correlation index coincides with Spearman's rank correlation, which exploits the natural ranking of education levels and is used in particular by Gihleb and Lang (2020). Equivalently, one can consider the χ^2

$$\chi^2(a,b,c,d) = I_{Corr}^2 = \frac{(r-mn)^2}{mn(1-m)(1-n)}$$

The minimum distance approach of Fernández and Rogerson (2001) and Abbott, Gallipoli, Meghir, and Violante (2019) constructs the convex combination of two extreme cases (random and perfectly assortative) that minimizes the distance with the table under consideration. Their index then is the weight of the perfectly assortative component in this combination:

$$I_{MD}(m,n,r) = \frac{r-mn}{n-mn}$$

This index also coincides, in the 2×2 case, with the "perfect-random normalization" of Liu and Lu (2006) and Shen (2019).

Finally, Eika, Mogstad, and Zafar (2019) measure marital sorting between men of education level I and women of education level J as "the observed probability that a husband with education level I is married to a wife with education level J, relative to the probability under random matching with respect to education". In the 2×2 example, this gives the likelihood ratio:

$$I_L(m,n,r) = \frac{r}{mn}.$$

3.1.2 An Axiomatic Approach

To help choose from this bewildering variety of indices, Chiappori, Costa-Dias, and Meghir (2020) introduce the requirement of "Weak Perfect Positive Assortative Matching". To define it, consider Table 1 and suppose that $m = n = r$. In other words, there is an equal number of educated men and women; matching is perfectly assortative, in the sense that both Educated and Uneducated people *exclusively* marry their own kind. The WPPAM condition imposes that no other table can be ranked as strictly more assortative than this matching.

Table 2: *

Changes in Assortative Matching—An Example

Table $A = (.03, .1, .1)$

$_w\backslash^h$	E	U
E	.03	.07
U	.07	.83

Table $B = (.5, .5, .5)$

$_w\backslash^h$	E	U
E	.5	0
U	0	.5

Chiappori, Costa-Dias, and Meghir (2020) show that all but one of the criteria just described do satisfy this requirement. The exception is the likelihood ratio index I_L, as shown by the comparison of the two cohorts of Table 2. Between cohorts A and B, the number of Educated people has drastically increased (from 10% to 50%). In cohort A, matching is positive assortative in the usual sense (more people on the diagonal than would obtain under random matching); yet, 70% of Educated individuals marry an Uneducated spouse. Cohort B, on the contrary, displays perfect assortative matching: all Educated men marry Educated women, and conversely. All criteria excepted I_L conclude that B displays more assortativeness than A. Yet we have

$$I_L(A) = 3 \quad \& \quad I_L(B) = 2$$

which would lead Eika, Mogstad, and Zafar (2019) to conclude that assortativeness has **decreased** from A to B[52].

3.1.3 A structural interpretation

Besides its axiomatic properties, the odds ratio admits a direct, structural interpretation in a frictionless framework. Specifically, starting from the logit model described in Section 2.3, it is easy to show that Table 1 is generated by any logit model such that

$$\frac{1}{2}\left(S_{EE} + S_{UU} - S_{EU} - S_{UE}\right) = \ln\left(\frac{r\left(1 + r - m - n\right)}{\left(n - r\right)\left(m - r\right)}\right).$$

The left-hand side of this equation is one-half of what Chiappori, Salanié, and Weiss (2017) called the *supermodular core* of the marital surplus, which is, in that context, a direct measure of the preference for assortative matching on education; while the right-hand side is simply the odds ratio defined above.

The main advantage of this structural interpretation is that it provides a theoretically clean methods for disentangling the mechanical impact of variations in marginal distributions from more fundamental changes in the economic gains from homogamy. In the frictionless context, the latter are fully summarized by the surplus matrix S and its supermodular core. When comparing two tables, one can readily compute the equilibrium that would obtain with the surplus of the first and the marginals of the second, thus obtaining clear counterfactual simulations.

[52] This is related to the fact that I_L violates a standard requirement of the statistical literature on measures of association in the case of paired attributes—Edwards (1963)'s Marginal Independence condition. See Chiappori, Costa-Dias, and Meghir (2020) for a detailed discussion.

3.1.4 The n-by-n case

Extending the previous analysis to $n \geq 2$ education categories raises an additional problem: homogamy may not be uniform across the various categories. For instance, matching patterns can be assortative at the top of the distribution but not at the bottom (or conversely). Moreover, assortativeness may increase over time among more educated groups while declining among the less educated ones.

For this reason, summarizing the global evolution of assortativeness can generate misleading results. It will generally be sensitive to the choice of aggregation over several groups (or education levels in our running example)[53]. In particular, statements like "educational homogamy did not change over a given period" should be handled with care, at least when they are based on a single indicator. A constant index may reflect the absence of any variation; or it may result from offsetting changes operating in different subsets of the population.

Chiappori, Costa-Dias, and Meghir (2020) discuss several possible strategies for extending the previous analysis to the $n \times n$ case. One may adopt a partial view and concentrate on a specific subset of categories—e.g. changes in homogamy among the more educated groups. Alternatively, one could concentrate on one particular category and merge all others into an "everyone else" class. In this case, variations in assortativeness in different margins could be concealed by the aggregation of many categories into a single one.

3.2 Abortion law and marriage market outcomes

In the first application of separable models of matching, Choo and Siow (2006) investigated the effect of the *Roe v. Wade* 1973 Supreme Court ruling on the marriage market in the US. At the time of the ruling, different states varied widely in how they regulated abortion. In what Choo and Siow (2006) call the "non-reform" states, *Roe v. Wade* had no effect on state law; in "reform" state it made abortion more accessible. The paper fits the logit model of Section 2.3 to Census and Vital Statistics data. They compute four sets of nonparametric estimates of the marital surplus S: on non-reform and reform states, before and after the 1973 ruling. In each case the type consists of the age of each individual.

These four sets of estimates allow Choo and Siow (2006) to evaluate the effect of *Roe v. Wade* on the gains to marriage u_x and v_y. Their findings suggest that the partial legalization of abortion in the reform states is partly responsible for the drop in marriage rates over the 1970s, as well as for an increase in the age of marriage. One possible explanation

[53] See for instance Gihleb and Lang (2020).

is that the legalization of abortion allowed some young adults to avoid a "shotgun marriage".

3.3 The Marital College Premium

Several recent contributions have used a structural approach to investigate how the marriage market interacts with human capital investments. Consider the demand for higher education. Over the last decades, the college premium has surged in many labor markets, boosting the returns to investments in college education and beyond. Not surprisingly, the proportion of *women* with a graduate degree has vastly increased; however, the proportion of men has stagnated at best. Chiappori, Iyigun, and Weiss (2009b) suggest that this striking asymmetry may originate in the marriage market. Define the "marital college premium" as the difference between the expected gains of college-educated individuals on the marriage market and those of less-educated individuals. Note that this marital premium comes over and above the labor market premium. Chiappori, Iyigun, and Weiss (2009b) show how the evolution of marital patterns over the period is compatible with a decrease (resp. increase) in the male (female) premium. The intuition is simple. When few women were educated, many uneducated women "married up" and not being educated did not hurt women's marital prospects much. As more and more women go to college (or beyond), those who do not face tougher competition on the marriage market[54]. Symmetrically, less-educated men become more likely to marry a college-educated woman.

This idea was taken to 30 years of data on the US marriage market by Chiappori, Salanié, and Weiss (2017). They start by fitting a logit model of the following form:

$$\tilde{S}(i,j) = S_{IJ} + \alpha_I^c + \beta_J^c + \zeta_J^c(i) + \xi_I^c(j)$$

where woman i and man j belong to cohort c and have education levels I and J. This model allows for arbitrary changes in the marriage rates of the different types of men and women; but it restricts the supermodular core to be constant over these 30 cohorts. It is strongly rejected for the white population (although it is not for African-Americans). Next, they allow for a trend:

$$S_{IJ}^c = a_{IJ} + b_{IJ} \times c. \tag{32}$$

The fit with actual patterns is considerably improved; moreover, the matrix $B = (b_{IJ})$ is supermodular, indicating stronger preferences for

[54] Even though the stable matching is unique, its qualitative features may exhibit large responses to minor changes in the fundamentals, an effect reminiscent of the social interaction literature.

assortative matching over time[55].

Why such an evolution? In the authors' explanation, investments in the human capital of children play a central role. Given the spectacular rise of college and post-college premium in the US, the time spent by parents with their children has significantly increased, particularly among more educated households. If, as empirical studies clearly suggest, the parents' own human capital is an important input in the process, theory predicts that assortative matching should increase in response. In the long run, this mechanism may mechanically amplify inequality cohort after cohort, generating the "inequality spiral" described in Chiappori (2017).

Low (2021) reaches similar conclusions from a more complex framework, in which men only differ by their innate ability whereas women differ by two traits: their innate ability *and* their fertility. Women may boost their innate ability by graduate education; while this increases their income, it reduces the time available to have a child. When returns to human capital were small and the loss of fertility was perceived as costly, the stable matching could exhibit non-monotonic patterns: top-earning men preferred less skilled but more fertile women. As returns to human capital increased and desired fertility fell, the stable matching switched to assortative matching on human capital. Low shows that the evolution of the US marriage market over the last decades can be interpreted as a shift of this type.

3.4 Household formation and dissolution

3.4.1 Divorce in a frictionless matching framework

In an early contribution, Peters (1986) used the generalization of unilateral divorce in the US to test two models of divorce. She showed that if information is symmetric at the time of divorce, divorce laws should have no effect on divorce rates. On the other hand, with asymmetric information a shift to unilateral divorce should increase divorce. She found that it did not.

Voena (2015) investigates the impact of divorce laws on household behavior, and particularly on the distribution of resources within the couple. In her model, a non monetary, "quality" shock follows a random walk stochastic process, so that the taste for the current marriage displays persistence. Economies of scale in (private) consumption generate marital surplus. The framework is explicitly dynamic, in a Limited Intrafamily Commitment (LIC) setting; human capital accumulation,

[55] Eika, Mogstad, and Zafar (2019) reach the opposite conclusion; see our discussion of their indicator in Section 3.1, however.

which depends on participation decisions at each period, determines the wage dynamics. Finally, the allocation of resources after divorce is dictated by the existing legislation.

Voena's empirical strategy exploits the shifts from mutual consent to unilateral divorce in several US states, and the panel dimension of the Panel Study of Income Dynamics. As predicted by theory, the impact of the reform crucially depends on another dimension of the legislation: the treatment of the couple's assets upon divorce. In those states where courts tend to equally divide assets between spouses, the switch to unilateral divorce should unambiguously affect the intra-household allocation of power by enhancing the bargaining position of the less wealthy spouse (usually the wife); indeed, data reveal a significant change in savings and labor supply in exactly the predicted direction. This should not happen when partners keep their initial assets upon dissolution; data also confirm this prediction.

In the same line, Chiappori, Iyigun, Lafortune, and Weiss (2017) analyze the effects of a reform granting alimony rights to cohabiting couples in Canada. A simple matching model under TU predicts that changes in alimony laws would affect existing couples and couples-to-be differently. In existing couples, it benefits the intended beneficiary. For couples not yet formed, however, it generates offsetting intra-household transfers and lower intra-marital allocations for the lower-income partner. The empirical analysis confirms these predictions: the right to petition for alimony led women to lower their labor force participation in existing couples, but not among couples that started cohabiting after the reform.

Legislation affecting the division of assets upon the death of a spouse may have long term effects on matching patterns, including among young individuals. Persson (2020) analyzes a 1989 reform in Sweden that abolished the "survivor insurance" by which the surviving spouse was granted a lifetime annuity. She shows that even though the financial impact of the reform would not be felt before several decades, it immediately affected marital patterns. By reducing marital surplus, it decreased the number of marriages and increased the steady state rate of separation from cohabiting unions. Some cohabiting couples, however, were given the opportunity to benefit from the old system if they married before a given deadline. This resulted in a considerable surge in marriages in previously cohabiting couples. One would expect the divorce rate to be higher for the affected couples than for the rest of the population; the data confirm this prediction. Finally, as survivors' insurance de facto subsidized matches with highly unequal earnings, its abolition caused an increase in assortative matching.

Lise and Yamada (2019) showed that a positive shock to a spouse's wage leads her household to shift the Pareto weights in her favour, especially if the shock is large enough and both partners work. They also found that divorce is often preceded by large adjustments on the weights.

3.4.2 Search models, divorce and (re-)marriage

We now turn to search models of the marriage market. We described the pioneering contribution of Shimer and Smith (2000) in Section 1.4.2. Their focus was on the existence and property of a steady-state equilibrium, and their model lacked several realistic features. In particular, divorce was a purely random event, unrelated to the quality of the match or any shock to earnings. They also did not account for heterogeneous attitudes towards divorce among individuals. Recent empirical applications have enriched the model in both of these dimensions.

Goussé, Jacquemet, and Robin (2017) construct and estimate on British data a model of marriage formation and dissolution aimed at explaining the dynamics of household behavior, in particular labor supply and home production time inputs. Their model introduces two important innovations. First, divorce is explicitly modeled as a steady-state phenomenon. When hit by a negative "bliss shock", spouses may either separate[56] or renegotiate how resources and duties are allocated within the household[57]. Second, the authors allow for behavior to be influenced by family values, which are heterogeneous among individuals. In particular, they present counterfactual simulations of an economy where all individuals have "liberal" values; they show that the marriage rate would decline, and married women would increase labor market participation very substantially.

Ciscato (2019) estimates a related model on US data. In his framework, spouses are able to insure each other against wage shocks. However, in the absence of full commitment, both wage and love shocks can trigger divorce; then agents are free to look for a new spouse, but their marriage prospects deteriorate as they get older. The model, estimated for two separate periods, the 1970s and the 2000s, replicates the cross-sectional marriage patterns, the life-cycle marriage and divorce patterns, and the female labor supply patterns. Up to a third of the decline in the share of married adults between the 1970s and the 2000s appears to be due to changes in the wage distribution.

A recent contribution by Shephard (2019) departs from the previous body of works in several respects. He considers an overlapping generation model in which individuals, at each period, can meet at most one

[56] In which case they can choose to return to the marriage market.

[57] The model assumes no commitment: *any* (monetary or non monetary) shock affecting the household triggers renegotiation.

potential spouse of all marriageable cohorts; should marriage occur, the match quality evolves stochastically. Importantly, Shephard assumes limited commitment à la Mazzocco (2004). This allows for a significant amount of risk sharing within the couple: while agents cannot commit not to divorce, marriage contracts are second best efficient ex ante. Pareto weights are only renegotiated when the participation constraint of one spouse becomes binding. Finally, the presence of several cohorts that may intermarry allows Shephard (2019) to analyze topics like the evolution of age at first marriage or of the marital age gap—aspects that are influenced by the economic environment and that in turn impact household behavior. For instance, Shephard finds that the significant increase in women's relative earnings since the 1980s increased female employment and the age at first marriage for women, while reducing male employment and the marital age gap.

3.4.3 Marital migrations

Most of the literature considers "closed" marriage markets. A particularly interesting situation appears when individuals can find partners outside their initial markets, possibly at some additional cost. The development of "marital migrations" provide an important example.

Over the recent decades, Asia has witnessed a surge in transnational marriages, with men in Singapore, South Korea and Taiwan marrying women from poorer countries such as Vietnam. Ahn (2020) studies the consequences of the emergence of match-making intermediaries on marriage migrations from Vietnam to Taiwan. Between 1995 and 2000, the number of Vietnamese women marrying a Taiwanese husband surged from a few hundred to almost 15,000 per year; after a visa tightening policy was implemented in 2004, however, the yearly number dropped to less than 5,000. Ahn shows that the marital patterns closely follow theoretical predictions. Cross-marrying Taiwanese men are selected from the middle of the Taiwanese socioeconomic status distribution, while Vietnamese women come from the top of the Vietnamese one; and the costs of cross-matching affect this selection in the predicted manner. Even though within-borders marriage remains minor in Vietnam, its impact on the allocation of bargaining power within couples in the regions most affected turns out to be large. Using a difference-in-difference approach, Ahn documents a significant decrease (resp. increase) in private consumption of male- (resp. female-) exclusive goods (e.g., smoking vs jewels).

3.5 Personality traits and marriage

The introduction of continuous types in separable matching models by Dupuy and Galichon (2014) opened the door to including a much broader set of types in empirical analysis. They demonstrated this by using the Dutch DNB Household Survey, which contains very rich information: in addition to the standard sociodemographic variables, it includes health, height and weight of each member of the household, as well as answers to a questionnaire on personality and on risk attitudes. They recoded the answers to the questionnaire into six continuous scales that reflect the "Big Five" personality traits[58] and risk aversion.

After adding age, education, height and BMI, this gives Dupuy and Galichon (2014) a total of eleven variables. They input them into a quadratic surplus S_{xy} and they estimate its affinity matrix[59]. They reject the hypothesis that the affinity matrix has reduced rank. In order to elucidate its most "salient" features, they compute its principal components. The first one, which explains 28 percent of the variance of S_{xy}, loads heavily on education. The second one has high loadings for personality traits, most notably emotional stability for men and conscientiousness for women; it explains 17 percent of the variance.

The same methodology is used by Ciscato and Weber (2020) to describe mating patterns in the USA from 1964 to 2017 and to measure the impact of changes in marital preferences on between-household income inequality. Analyzing matching by education, wage, age and race, the authors find that, after controlling for other observables, assortative mating has become stronger, with a significant impact on inequality: if mating patterns had not changed since 1971, the 2017 Gini coefficient between married households would be 6% lower.

3.6 Same-sex marriage

The study of the differences between same-sex and opposite-sex couples has become an active area of research. It has generally found that same-sex couples exhibit less specialization than opposite-sex couples (Jepsen and Jepsen, 2015). Oreffice (2011) also finds that decision power skews towards the younger partner in same-sex couples. Gay and lesbian couples are quite different: gay couples are more specialized, and lesbian couples are more concerned with fertility. Aldén, Edlund, Hammarstedt, and Mueller-Smith (2015) show for instance that lesbian couples became more common after Sweden extended joint parenting rights to

[58] Conscientiousness, Extraversion, Agreeableness, Emotional stability, and Autonomy.

[59] See Section 2.4.4.

same-sex couples.

Ciscato, Galichon, and Goussé (2020) built on Dupuy and Galichon (2014) to compare marital surplus functions for homosexual and heterosexual couples. They model gay men, lesbians, and heterosexuals as matching on three separate markets—that is, they take sexual orientation as immutable. Their empirical application uses American Community Survey data from California in the five years that followed its legalization of same-sex marriage. Ciscato, Galichon, and Goussé (2020) estimate affinity matrices with age, education, race and wages as types. Their results suggest that compared to different-sex couples, same-sex couples of both genders have a less pronounced preference for assortative matching on age and race. The preference for assortative educational matching is stronger for lesbians than for either gay men or heterosexuals.

4 Conclusion

The economics of mating is an extremely active field that keeps attracting original contributions and ideas. Predicting the path of future innovations is notoriously hard. Still, some research directions seem particularly promising to us. One is the joint estimation of matching patterns and household behavior. The empirical analysis of household behavior has experienced spectacular progress over recent decades. Still, most such investigations take existing families as given. Matching models, on the other hand, make it clear that household formation and dissolution are endogenous processes that must be understood and explicitly taken into account. Not only do the corresponding mechanisms explain couples; they also codetermine the nature of the intrahousehold relationship, and particularly the allocation of decision power within it.

Conversely, studying post-marital behavior can significantly enrich our understanding of matching patterns. Direct information on the nature or the magnitude of the surplus would help identify the primitives of the model. The empirical analysis of post-marital behavior can provide this type of information; gains from marriage translate into higher well-being for both spouses, and we should be able to recover these gains from the observation of consumption and labor supply decisions. Such an approach would allow to fully exploit the complementarity between matching theory on the one hand and collective models of household behavior on the other hand. This path was initially introduced by Chiappori, Costa-Dias, and Meghir (2018) in a Transferable Utility context.

Three remarks are in order. First, given the limitations of the Transferable Utility context (as discussed above), extending this approach to an Imperfectly Transferable Utility framework seems a promising objective. Second, when studying household behavior, special emphasis should be put on activities that are most likely to generate marital gain, such as investments on children. In particular, the technology of human capital production and accumulation has important implications on the matching process, particularly on preferences for homogamy. The reader is referred to Chiappori, Costa-Dias, Meghir, and Xiao (2021) for an investigation along these lines. Third, all approaches described in this survey represent consumption or labor supply decisions as continuous (and generally differentiable) functions of prices and incomes. An alternative and by now well understood approach relies on a revealed preferences perspective. This explicitly recognizes the discrete nature of existing data sets; it translates theoretical restrictions into a set of inequalities involving observable data points. In a recent contribution, Cherchye, Demuynck, Rock, and Vermeulen (2017) add to the revealed preference framework a set of additional constraints that reflect stability in matching.

One advantage of the revealed preference method is that it does not require—but it can accommodate—specific assumptions on preferences or the nature of the transfers. A well-known issue is that it the interpretation of the violations of inequalities is not straightforward. Cherchye, Demuynck, Rock, and Vermeulen (2017) introduce a match-specific "stability index" to quantify the violations; it is related to the income loss that must be associated with separation in order to rationalize away blocking couples. The conclusions of their study are very informative; they find for instance that the constant share assumption (by which the fraction of resources received by each spouse does not vary with income) is not rejected[60].

Remarriage constitutes a second promising direction for future research. While modeling divorce is fairly easy under the assumption that divorcees remain single for the rest of their life, introducing re-matching opportunities is more difficult. The benefits from remarriage depend on the characteristics of divorcees; conversely, divorce decisions are endogenous and are partly driven by conditions on the (re-)marriage market. The existence of commodities that remain public goods for ex-spouses even after their divorce further complicates the picture, as do the various constraints that limit individuals' ability to commit on future behavior. All in all, much remains to be understood regarding these questions.

Ultimately, a better understanding of issues related to household formation, dissolution and decision processes is an indispensable step.

[60] Browning, Cherchye, Demuynck, Rock, and Vermeulen (2021) extend Cherchye, Demuynck, Rock, and Vermeulen (2017) to allow for unobservable maych quality under some restrictions on preferences.

Any assessment of the consequences of a policy over the long term cannot ignore its impact on these demographic determinants. To take only one example, the short-term effect of a tax reform is a change in individual savings or labor supply. In the long run, however, the indirect impact on incentives to marry and to invest into human capital (whether one's own or that of the children) could well matter more. Neglecting these general equilibrium consequences may result in dramatic misconceptions. Similarly, while changes in assortativeness may have a direct impact on cross-sectional inequality, their long term consequences in terms of investment in human capital are probably more significant. Changing preferences for assortativeness, and especially the desire to invest into children's education in the most efficient way, could have far-reaching consequences in terms of global inequality.

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